AN UNIFIED APPROACH FOR TWO-STAGE TIME DOMAIN MODAL IDENTIFICATION

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SUMMARY: .A common mathematical framework is established for a unified two-stage time domain (TD) modal identification, based on a formula of modal decomposition of the time response function (TRF), represented as impulse response function, free decay response, or correlation function, as well as data correlation of the TRF. Possible implementations for the TD two-stage modal identification, which cover a variety of well-known techniques, and numerical consideration, are summarized to provide better understanding of the different techniques, and guidelines for effective applications. Major issues, e.g. measured data selection, structural mode sorting and estimation uncertainty analysis. As well as further improvements of this seemly-matured two-stage time domain modal identification are discussed.

KEYWAORDS: Modal Identification, Modal Parameters, Impulse Response Function (IRF), Free Decay Function (FDF), Correlation Function (CF), Least Squares Estimation (LSE)

INTRODUCTION

System identification is of great importance in many engineering area. Modal identification, developed in vibration engineering, is to build modal model, and therefore reduces the problem to modal parameter estimation. One of the major features of modal identification is normally making use of Frequency Response Functions (FRFs) or Time Response Functions (TRFs), instead of input/output data in time domain (TD) or frequency domain (FD) directly.

Two-stage TD modal identification is a unified approach for estimation of modal parameters from time response function (TRF) based on common formulation of modal decomposition of TRF. Here TRF is defined to represent unit Impulse Response Function (IRF), Free Decay Response (FDR), Random Decrement (RDD) signature, as well as Cross Correlation Function (CCF). IRF is the counterpart of FRF in FD, and can be calculated via inverse FFT. Cross Correlation Functions (CCFs) are normally calculated from cross Power Spectrum Density (PSD) via inverse FFT in the output only cases. RDD signature was explained as free decay of the system at first [1], [2] and then proved as correlation function of the response, and can be computed through many ways from random response of the system [3], [4]. FDR, as well as CCF, can also be expressed as a sum of exponentially decaying sinusoids. Each decaying sinusoids has a damped natural frequency and damping ratio that is identical to the one of the corresponding structural mode. FDRs can be measured either by impulse excitation or sudden termination of board band force excitation...

Historically, Complex Exponential algorithm is one of the earliest multiple degree-of-freedom modal identification techniques in TD (1974), and improved via Least Squares solution and expended for Single Input Multi-Output (SIMO) case as Least Squares Complex Exponential (LSCE) algorithm in 1977 [5]. In the same year, well-known modal identification procedure—Ibrahim Time Domain (ITD) was developed. ITD was formulated as SIMO technique at very beginning [6]. However, it can be easily extended for Multi-Input Multi-Output (MIMO) application [7], though. The first MIMO version of modal identification algorithm, which was an important breakthrough in experimental modal analysis was the technique called as Polyreference Complex Exponential (PRCE), as an extension of LSCE, developed in 1982 [8]. Eigensystem Realization Algorithm (ERA), based on system realization theory in linear system analysis, was derived in 1984 [9] from state-space model, which is often utilized in control engineering. To reduce the influence of noise contamination in the TRF data, an improved PRCE called Improved Polyreference technique (IPCE), which makes use of correlation of the TRF data, was developed in 1987[10]. Data correlation version of ERA, as ERA/DC [11], followed in 1988. Due to data correlation, which acts as a correlation filter, the order of the model to be identified can be reduced and identification accuracy is increased.

In early 1990s, Natural Excitation Technique (NExT) was proposed for modal identification from output data only in the case of natural excitation [12]. NExT actually is an idea that suggests using cross correlation function (CCF) of the random response of the structure under natural excitation, which often is broadband random process. All the MIMO versions of TD modal identification procedures mentioned above can be used as NExT. Based on this idea, the two-stage modal identification techniques cannot only be adopted in traditional modal analysis, but also for ambient or operational modal analysis.

A unified two-stage modal identification approach in TD is proposed during a re-visit to modal identification developed in the last three decades. The unified approach is based on the formula of modal decomposition of TRF, and can cover all above-mentioned TD modal identification algorithms. Under the same common mathematical framework, we can have better understanding of all different implementation, the features, advantages and disadvantages of different algorithms. Numerical accuracy or/and efficiency can be improved via comparisons of different implementation. As a part of the unified approach, implementation and numerical considerations, as well as major issues for the two-stage TD modal identification will be discussed in some details in the paper. A separate paper will be published to summaries TRF estimation for the two-stage modal identification.

1. COMMON FRAMEWORK FOR MODAL IDENTIFICATION

The two-stage TD modal identification approach is based on a common mathematical model as the modal decomposition of the Time Response Function (TRF)

$$h_k = \Phi e^{\Lambda k \Delta t}\Gamma^T = \Phi Z^k \Gamma^T$$
$$[Z] = diag[z_r] = e^{\bar{\Lambda}\Delta t}, \ \bar{\Lambda} = \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda^* \end{bmatrix} \qquad (1\text{-}1)$$

Where h_k represents kth sampling point of the TRF, and is an $m \times l$ matrix estimated from m response measurements with respect to l excitation or reference locations. Φ, Γ is $m \times N$ complex mode shape matrix and $l \times N$ modal partition factor (MPF), respectively. Λ is an $N \times N$ complex modal frequency matrix, from which modal frequency w_r and damping ration z_r can be simply calculated from the following formulation

$$l_r = -a_r + jb_r = -V_r w_r + jw_r\sqrt{1-V_r^2}$$
$$l_r^* = -a_r - jb_r = -V_r w_r - jw_r\sqrt{1-V_r^2} \qquad (1\text{-}2)$$

In the analytical modeling, N is the number of degree-of-freedoms (DOFs) of the linear time-invariant dynamic system:

$$M\ddot{x} + D\dot{x} + Kx = bf(t)$$
$$y(t) = cx(t) \qquad (1\text{-}3)$$

In the above equations $f(t)$, $y(t)$ and $x(t)$, are input, output and displacement state vector with dimensions of l, m and N, respectively. M, K and D are $N \times N$ mass, stiffness and damping matrices, respectively. b, c is $N \times l$ input influence matrix and $m \times N$ output influence matrix, respectively. When the TRF decomposition formula is utilized for modal identification, N will stand for the number of modes.

The common mathematical framework of the two-stage TD modal identification can be described based on the basic mathematical model as follows. Making p block row shift of the basic Eqn 1-1 when the number of measurements is smaller than modes to be identified yields

$$\begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_p \end{bmatrix} = \begin{bmatrix} \Phi \\ \Phi Z \\ \vdots \\ \Phi Z^p \end{bmatrix} \Gamma^T \qquad (1\text{-}4)$$

Or in compact form

$$\tilde{H} = \tilde{\Phi}\Gamma^T \quad where \tilde{H} = \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_p \end{bmatrix}, \ \tilde{\Phi} = \begin{bmatrix} \Phi \\ \Phi Z \\ \vdots \\ \Phi Z^p \end{bmatrix} \qquad (1\text{-}5)$$

Assume there are N-pair of complex modes, and mp=2N, therefore the matrix $\tilde{\Phi}$ has full row rank of mp. Since the $m(1+p) \times mp$ matrix $\tilde{\Phi}$ has m more rows than columns, there must exist a $m \times m(1+p)$ matrix $\tilde{A}$ so that

$$\tilde{A}\tilde{\Phi} = 0 \qquad (1\text{-}6)$$

From Eqn 1-4 we can obtain the following equation

$$\tilde{A}\tilde{H} = 0 \qquad (1\text{-}7)$$

Partitioning matrix $\tilde{A}$ into (1+p) blocks $m \times m$ matrices yields

$$\begin{bmatrix} A_0 & A_1 & \cdots & A_p \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_p \end{bmatrix} = 0 \qquad (1\text{-}8)$$

An multivariable Autoregration (AR) equation is therefore obtained, which can be normalized as A_p=I and written as

$$\begin{bmatrix} A_0 & A_1 & \cdots & A_{p-1} \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{p-1} \end{bmatrix} = -h_p \qquad (1\text{-}9)$$

In order to solve AR coefficient matrix $m \times m$ matrix A_i (i=0,1,↑p-1), an over determined equation could be arranged via block column shift of matrix $\tilde{H}$ as follows

$$\begin{bmatrix} A_0 & A_1 & \cdots & A_{p-1} \end{bmatrix} \begin{bmatrix} h_0 & h_1 & \cdots & h_{q-1} \\ h_1 & h_2 & \cdots & h_q \\ \vdots & \vdots & \ddots & \vdots \\ h_{p-1} & h_p & \cdots & h_{p+q-2} \end{bmatrix} = -\begin{bmatrix} h_p & h_{p+1} & \cdots & h_{p+q-1} \end{bmatrix} \qquad (1\text{-}10)$$

The matrix $\lfloor A_0 \ A_1 \cdots A_{p-1} \rfloor$ can, therefore, be estimated via Least Squares (LS) technique, when q is selected to satisfy the relation of lq> mp.

Having the estimates of the system characteristics matrix $\lfloor A_0 \ A_1 \cdots A_{p-1} \rfloor$, we can build its relation with modal parameters contained in Z and Φ via block matrix form of Eqn 1-6

$$\begin{bmatrix} -A_0 & -A_1 & \cdots & -A_{p-1} \end{bmatrix} \begin{bmatrix} \Phi \\ \Phi Z \\ \vdots \\ \Phi Z^{p-1} \end{bmatrix} = \Phi Z^p \qquad (1\text{-}11)$$

A standard eigenvalue problem can be formulated by constructing a companion matrix of the AR coefficient matrices

$$\begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -A_0 & -A_1 & -A_2 & \cdots & -A_{p-1} \end{bmatrix} \begin{bmatrix} \Phi \\ \Phi Z \\ \Phi Z^2 \\ \vdots \\ \Phi Z^{p-1} \end{bmatrix} = \begin{bmatrix} \Phi \\ \Phi Z \\ \Phi Z^2 \\ \vdots \\ \Phi Z^{p-1} \end{bmatrix} Z \tag{1-12}$$

From the eigenvalue matrix Z of the above eigensolution, the complex modal frequency matrix Λ=diag[l_r] can be calculated by the following formulas

$$b_r = \frac{1}{\Delta t} \tan^{-1} \left[\frac{\Re(z_r)}{\Im(z_r)} \right], \quad a_r = \frac{1}{2\Delta t} \ln\left([\Re(z_r)]^2 + [\Im(z_r)]^2 \right) \tag{1-13}$$

$$where \quad l_r = -a_r + jb_r, \; z_r = \Re(z_r) + j\Im(z_r), \quad r = 1,2,\cdots N$$

Modal frequencies and damping ratios can then be computed as mentioned in previous section. The modal matrix can be obtained from the first m lines of the eigenvector matrix.

To obtain full sets of modal parameters, the MPF matrix Γ should be calculated. Again, over determined equation could be formulated to compute MPF matrix from measured TRF data and estimated $\tilde{\Phi}$ by LS solution

$$\begin{bmatrix} h_0 \\ h_1 \\ \vdots \\ h_{s-1} \end{bmatrix} = \begin{bmatrix} \Phi \\ \Phi Z \\ \vdots \\ \Phi Z^{s-1} \end{bmatrix} \Gamma^T \tag{1-14}$$

Where ms>2N is required to form LS solution.

Another way of manipulating the basic equation is to make block column shift instead of row shift

$$\begin{bmatrix} h_0 & h_1 & \cdots & h_q \end{bmatrix} = \Phi \begin{bmatrix} \Gamma^T & Z\Gamma^T & \cdots & Z^q \Gamma^T \end{bmatrix} \tag{1-15}$$

Where lq=2N is assumed. In light of the exact same argument, we have the following AR equation

$$\begin{bmatrix} \Gamma^T & Z\Gamma^T & \cdots & Z^q \Gamma^T \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ \vdots \\ A_q \end{bmatrix} = 0 \tag{1-16}$$

In this case, the dimension of the AR coefficient matrix A_i is $l \times l$ (i=0, 1,↑q). Three similar equations for modal identification can be derived following the same procedure as follows.

(1) An over determined equation for solving AR coefficient matrices]

$$\begin{bmatrix} h_0 & h_1 & \cdots & h_{q-1} \\ h_1 & h_2 & \cdots & h_q \\ \vdots & \vdots & \ddots & \vdots \\ h_{p-1} & h_p & \cdots & h_{p+q-2} \end{bmatrix} \begin{bmatrix} A_0 \\ A_1 \\ \vdots \\ A_{q-1} \end{bmatrix} = - \begin{bmatrix} h_q \\ h_{q+1} \\ \vdots \\ h_{p+q-1} \end{bmatrix} \tag{1-17}$$

Where mp should be larger than lq to ensure LS solution.

(2) Standard eigenvalue problem for Modal frequencies, damping ratios and MPFs

$$\begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -A_0^T & -A_1^T & -A_2^T & \cdots & -A_{q-1}^T \end{bmatrix} \begin{bmatrix} \Gamma \\ \Gamma Z \\ \Gamma Z^2 \\ \vdots \\ \Gamma Z^{q-1} \end{bmatrix} = \begin{bmatrix} \Gamma \\ \Gamma Z \\ \Gamma Z^2 \\ \vdots \\ \Gamma Z^{q-1} \end{bmatrix} Z \tag{1-18}$$

(3) Linear equation for modal matrix Φ,

$$\begin{bmatrix} h_0 & h_1 & \cdots & h_{s-1} \end{bmatrix} = \Phi \begin{bmatrix} \Gamma^T & Z\Gamma^T & \cdots & Z^{s-1} \Gamma^T \end{bmatrix} \tag{1-19}$$

In order to compute the modal matrix via LSE, ls>2N is required.

2. IMPLEMENTATION & NUMERICAL CONSIDERATIONS

It is clear from the previous section that implementation of the unified two-stage TD modal identification procedure consists of three steps.

1) Solving an over determined linear equations by LS technique to estimate AR coefficient block matrices with dimensions of $m\times m$, or $l\times l$, where m and l are number of outputs and inputs, respectively;
2) To calculate modal frequencies and dampings, as well as MPFs, or mode shapes, by solving standard eigenvalue problems;
3) To obtain mode shapes, or MPFs from LSE.

Many possible implementations and relevant numerical considerations are summarized as follows.

Assume p=2, it means that $m=N$, the equation for AR coefficient block matrix is written as

$$\begin{bmatrix} 0 & I \\ -A_0 & -A_1 \end{bmatrix}\begin{bmatrix} h_0\, h_1 \cdots h_{q-1} \\ h_1\, h_2 \cdots h_q \end{bmatrix} = \begin{bmatrix} h_2 & h_3 & \cdots & h_{q+1} \\ h_3 & h_4 & \cdots & h_{q+2} \end{bmatrix} \tag{2-1}$$

Or in compact form

$$AH_0 = H_1 \tag{2-1'}$$

System matrix A can then be solved by normal equation

$$A = [H_1 H_0^T][H_0 H_0^T]^{-1} \quad or$$
$$A = [H_1 H_1^T][H_0 H_1^T]^{-1} \tag{2-2}$$

This is exactly the ITD technique [6]. A double LS solution (DLS) was suggested as the average of the above two solutions [13].

When the number of response locations is less than the number of modes, i.e. $m<N$, the block row shift acts as "virtual measurements". The same LS solution is adopted in ITD technique. However, much smaller matrix containing AR coefficient matrices with dimensions of $m\times mp$ is required instead of $mp\times mp$ companion matrix A!

The direct utilization of normal equation is sometimes unstable due to the condition number of the data matrix. The more accurate way for LS solution is making use of orthogonal decomposition

$$H_0 = [R\,0][Q] = [R\,0]\begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = RQ_1 \tag{2-3}$$

Where H_0 is an $mp\times lq$ data matrix with $lq>mp$, Q is an $lp\times lp$ orthogonal matrix, and R is of dimension $mp\times mp$, or $2N\times 2N$, lower-triangular matrix with positive numbers in its diagonal. Orthogonal decomposition can numerically be implemented in one of the following methods: (1) Householder transformation, (2) Given method, or (3) improved Gram-Schmitt orthogonal method.

Another numerical technique for LS solution with ill-conditioned coefficient matrix is to adopt Singular Value Decomposition (SVD) of the TRF data matrix H_0

$$H_0 = USV^T = U_{ns}S_{ns}V_{ns}^T \tag{2-4}$$

Where S is diagonal singular value matrix U, V are orthogonal matrices consists of corresponding left and right singular vectors. S_{ns} is the sub-matrix with first ns dominate singular values, U_{ns} and V_{ns} are the partitions with corresponding singular vectors. The system matrix for ITD technique A_I can then be estimated as

$$A_I = H_1 H_0^+ = H_1 V_{ns} S_{ns}^{-1} U_{ns}^T \tag{2-5}$$

It is interesting to aware that for the ERA [9], its system matrix A_E is obtained as

$$A_E = S_{ns}^{-1/2} U_{ns}^T H_1 V_{ns} S_{ns}^{-1/2} \tag{2-6}$$

Obviously, there is a similarity transformation between these two system matrices

$$A_E = P^{-1} A_I P,\ where\ P = U_{ns} S_{ns}^{-1} \tag{2-7}$$

The eigenvalues calculated for two system matrices are exactly the same; the two eigenvector matrices are related by the linear transformation.

$$\tilde{\Phi}_E = P^{-1}\tilde{\Phi}_I = S_{ns}^{-1/2} U_{ns}^T \tilde{\Phi}_I \tag{2-8}$$

It is seen from Eqn1-12 that the complex modal frequencies and mode shapes are related to a high order ($mp\times mp$)

eigenvalue problem. Actually, only eigenvalue matrix Z needs to be computed. The eigenvector $\tilde{\Phi}$ can then be obtained from a linear equation with ($m\times m$) coefficient matrix

$$\left[\sum_{i=0}^{p} A_i z_r^i\right] \boldsymbol{f}_r = 0 \qquad (2\text{-}9)$$

where $z_r = e^{l_r \Delta t}, r = 1,2,\cdots 2n$. One numerically efficient way to solve this equation is making use of SVD, in this case actually is Eigenvalue Decomposition (EVD), of the coefficient matrix of the eigenvector $\boldsymbol{f}_r$

In the original presentation [6] the ITD is formulated as a single SIMO technique, and only un-scaled mode shapes can be calculated. However, it can actually be utilized in MIMO case. In order to obtain scaled mode shapes, the MPF matrix should be calculated as the third step of the two-stage modal identification procedure.

It can easily be observed that anther way of implementation of the two-stage modal identification is following the Eqn 1-16 ~ 1-18, which are the fundamental equations for the PRCE. When l=1, as a special case, the technique is further reduced to be Least Squares Complex Exponential (LSCE) algorithm, and can also be derived from applying Prony's algorithm [5]

3. MAJOR ISSUES IN TWO-STAGE MODAL IDENTIFICATION

The implementations of the two-stage TD modal identification procedures are basically matured. However, several issues should be carefully dealt with in order to make correct and accurate identification. The following are the major three of them.

(1) Data Selection

The two-stage modal identification is based on discrete time domain TRF (e.g. IRF, FDF or CCF) data. In order to identify all the modes in the frequency range of interest, the sampling theorem should be satisfied, i.e. the sampling rate should at least be larger than twice of the maximum value of the frequency band ($f_s \geq 2f_{\max}$). It means that the data spacing must be small enough. On the other hand, in order to model the slow decay of the low frequency modes well, there must be the TRF data refer to times after a significant amount of decay has occurred. This would result very large dimension of the data matrix for estimation of system characteristics matrix. However, it can bee seen from the Eqn 1-10 and 1-16, the actual maximum data number we need are $k_{\max}=p+q-1$, where p and q satisfy either $mp=2N$ and $lq>mp$, or $lq=2N$ and $mp>lq$. Normally, we have more TRF data available than needed. This fact provides the possibility of making choice of the measured data, including deletion of spurious data. Meanwhile careful and appropriate data selections are required for two-stage modal identification in order to reduce bias and variance errors.

(2) Modal Sorting

The number of structural modes is normally unknown before modal identification. Therefore, the assumed mode number N should always larger than true structural mode number n_s. In reality, even knowing the true mode number we still cannot let $N= n_s$. In order to accommodate all unwanted effects in the measured TRF data, e.g. input and response noise, leakage, residuals, non-linearity, etc., computational modes with number of n_c should be assigned to compensate these unwanted effects. Therefore, after the entire $N= n_s + n_c$ modes have been identified, the n_s structural modes should then be differentiated and detected, or the other n_c computational modes must be deleted.

There are three relevant methods to distinguish structural modes from computation ones, or modal sorting. First, the total number of modes should be defined before identification. Two indication techniques are available, which are based on Error Chart and Rank Chart, respectively. The error chart, as the plot of equation error versus model order is actually the by-product when making LS solution via QR decomposition. The rank chart can directly be obtained from SVD. Unfortunately, both of them work unfavorably in model order determination. The second method of modal sorting is making use of so called Stability Diagram. Stability diagram is a plot of possible modes, as modal frequency points, versus the model order. The structural modes are supposed to converge with the increasing of the model order. The convergence criterion can be the difference of subsequent two modal frequencies, or/and modal damping, and or/and mode shapes, represented by Modal Assurance Criterion (MAC). Experiences reveal that spurious modes cannot all be deleted based on such convergence criterion. The third method of distinguish structural modes from computational modes is to adopt specific modal indicator. Several modal indicators were proposed for different modal identification algorithms, e.g. Modal Confidence Factor (MCF) for ITD [14], extended latter for PRCE [15], Modal Amplitude Criterion (MAC) and Modal Phase Collinearity (MPC) [16], among others. An effort was made to extend all modal indicators mentioned above plus a Modal Partition Indicator (MPI) for unified two-stage modal identification [17]. It is believed that the combination of stability diagram and modal indicator(s) would be the best way for modal sorting.

(3) Identification Accuracy

The essence of the 2-stage modal identification approach is actually a Least Squares Estimation (LSE). The advantages of LSE are simple to implement and fast in computation. However, there is a serious drawback in LSE, i.e. it causes bias error in the estimates. In the LSE, a prediction error, or residual, $e_k = h_k - \hat{h}_k \quad (k = 1, 2, \cdots)$ is assumed when IRF, FDR and CCF are directly used as TRF to form the data matrix. It is well know that LSE would be unbiased only if the prediction error is white noise, i.e.

$$E[R_{ee}(i)] = 0, \quad for\ i \neq 0 \qquad (3\text{-}1)$$

In reality, e_k would never be such a white noise; even the system corrupted only by output/measurement noise and it is white! Therefore, bias error caused by colorness of the prediction error becomes one major problem in the LSE-based two-stage modal identification. There are two possible ways to overcome bias problem caused by LSE: one is to properly model the noise (noise modeling methods); the other is to eliminate bias error without noise modeling.

Actually noise modeling is not only to deal with measurement noise but also to compensate leakage, residuals and non-linearity. Many issues still remain to be explored. In the ambient modal identification cases, white excitation signals are normally assumed. However, colored excitation can be dealt with if a "shaping filter" is used to model the color noise. [18] Noise modeling will also bring lots of new problems, e.g. noise model selection, model order determination, and iteration convergence, etc. There are other methods available to reduce or eliminate bias error introduced by LSE, for example, the methods via Instrument Variable (IV), Double LS (DLS), Total LS (TLS) and LS with data correlation. IV procedure is also iterative, starting from LS estimates. DLS makes use of averaging of overestimates and under-estimate to reduce the bias error. TLS is base on seemly more reasonable error assumption that the noises are on the measurements in both sides of the equation, i.e. H_0 and $\tilde{H}_1$, instead of only on H_0 [10]. However, even TLS is utilize, bias will still result if e is colored noise! As mentioned in Section 1, the two-stage modal identification can be implemented using correlation filtering, or data correlation of TRF (IRF, FDR and CCF) [10], [11]. It can be proved that the LS estimation become theoretically unbiased, when correlation filter, or data correlation, is invoked with enough correlation data.

CONCLUDING REMARKS

1. A common mathematical framework for two-stage modal identification in time domain has been established. The unified two-stage modal identification is a typical global approach and can be applied to real world complex structures to identify full sets of modal parameters, including modal frequencies, damping ratios, scaled mode shapes and modal partition factors. It can also be utilized for operational modal analysis using free decay response, or correlation functions from output measurements only to obtain modal frequencies, damping ratios and un-scaled mode shapes.

2. There is variety of possible implementations for the TD two-stage modal identification, including well-known techniques, such as ITD, EITD, LSCE, PRCE, IPCE, ERA, ERA/DC, etc. Much better understanding for different techniques can be obtained under the common mathematical framework. Estimation accuracy and/or efficiency can be increased when numerical considerations are taken care of in the implementation.

3. Measured data selection, structural mode sorting, and estimation uncertainty analysis are three major issues for successful application of the two-stage TD modal identification. It will become more significant when dealing with array of measurement data from a complex structure, or very noisy data from operational or ambient measurements.

4. The common mathematical model for two-stage TD modal identification is the formula of modal decomposition of the time response function (TRF) which is defined as a sum of exponentially decaying sinusoids and represented as

impulse response function, (IRF), free decay response, or cross correlation function (CCF). The basic idea behind the two-stage TD modal identification is the least squares estimation (LSE), which has the advantages of simplicity and speed in implementation.

5. The two-stage TD modal identificatio n, after almost three decades development, seems matured. However, several issues are still remaining to be resolved. For examples, the influence of data selection to the estimation uncertainty (with both variance and bias errors); the more effective means or new modal indicators for structural modal sorting; further reduction of bias and variance error in the estimates while dealing with array of measurements from large complex structures, or very noisy data from operational or ambient measurements.

6. This paper summaries the two-stage modal identification in time domain. It' s counterpart in frequency domain, and one-stage modal identification based on state-space model making direct use of input/output or output data only will be discussed in other papers as the outcomes of re -visiting to the modal identification developed in last 30 yeaas.

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