DAMPING ESTIMATION OF ENGINEERING STRUCTURES WITH AMBIENT RESPONSE MEASUREMENTS

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ABSTRACT Damping plays a key role in dynamic response prediction, vibration control. Following the brief description of ambient vibration testing and measurements, three non-parametric damping estimation approaches for engineering structures are introduced: (1) time domain approach using Data Correlation (DC); (2) time-frequency analysis via Wavelet Transform (WT), and (3) a spatial domain approach based on Frequency Domain Decomposition (FDD). The major focus of the damping estimation approaches presented is on their capability in dealing with on-site full-scale testing of engineering structures using ambient response measurement. Computer simulations based on ambient response measurements and modal identification of a 15-story office building and a transmission tower were conducted to evaluate their performances of different approaches in dealing with closely spaced modes and noisy data. Major challenges in damping estimation are also discussed.

1. Introduction:

Damping is of great importance in dynamic design of engineering structures, especially for response prediction and vibration control, as well as in structural health monitoring during service. Dynamic response of a structure is determined by dynamics characteristics of the structure and external loads. Resonance plays a key role in the dynamic response especially for lightly damped structural system. Two critical parameters of resonance are resonance frequency and damping ratio, which are determined by mass, stiffness and damping characteristics of the system. In aerospace and civil engineering, structures are often subjected to broadband excitation, such as turbulence, wind gust, sea waves, traffic, etc. In this case, damping has critical inference on the dynamic response and is the main reason for excessive vibration in the structural system.

Structural vibration control becomes increasingly important due to increasingly stringent requirement in both aerospace and civil engineering. No matter the ways of implementation (passive, active or semi-active), the major two mechanisms of vibration control are energy dissipation and vibration isolation. The effects of energy dissipation enhancement may be achieved either by conversion kinetic energy to heat (damping), or transferring of energy to other device (dynamic absorbing). Innovative devices for supplement damping, such as dampers based on friction, material yield, viscoelastisity and viscous fluid, tuned mass and tuned liquid damper, as well as a variety of implementation of active control systems have been developed in recent years. Effectiveness of these devices should be verified and evaluated by damping estimation.

A dynamic structure can be damped by mechanisms with different internal and external nature. Generally, their mathematical description is quite complicated, and not suitable for theoretical analysis for complicated structures. In structural dynamics, damping can be described via viscous, hysteretic, coulomb and velocity squared model. Viscous damping occurs when the damping force is proportional to the velocity. Hysteretic damping, also known as structural damping, is associated with the hysteresis loop. It is frequency dependent and acts with a force that is proportional to displacement. Coulomb damping force caused by the friction opposes the direction of motion. Velocity squared damping occurs when a vibrating object is subjected to air resistance.

Hysteretic damping is limited to steady state vibrations. Coulomb and velocity-squared damping models are nonlinear, and mathematically inconvenient. Viscous damping is mathematically convenient because it results a linear second order differential equation. A transient decay of a viscously under damped system will decay exponentially. Therefore, from practical point of view, **equivalent viscous damping**, which models the overall damped behavior of the structural systems as being viscous, is often adopted in structural dynamics [1]. Even though, analytical modeling of damping is still very difficult, if not impossible, for real engineering structures. Therefore **damping estimation** via dynamic test and measurement becomes extremely important in structural dynamics, especially for the purpose of response prediction and vibration control.

Following the brief description of ambient vibration testing and measurements, three non-parametric damping estimation approaches for engineering structures are developed: (1) time domain approach using Data Correlation (DC); (2) time-frequency analysis via Wavelet Transform (WT), and (3) a spatial domain approach based on Frequency Domain Decomposition (FDD). The major focus of the approaches is on their capability in dealing with on-site full-scale testing of engineering structures using ambient response measurement. Computer simulations based on ambient response measurements and modal identification of a 15-story office building and a transmission tower were conducted to evaluate their performances of these approaches in dealing with closely spaced modes and noisy data. Major challenges in damping estimation are also discussed.

2. Ambient Dynamic Testing & Measurement

Traditional dynamic testing is conducted in the laboratory environment, where artificial controlled excitation force is applied and responses are measured. On-site testing is often required in aerospace and civil engineering. Artificial excitation can be applied in on-site testing, e.g. control surface or special winglet excitation in aircraft flight flutter testing; mechanical, electric or hydraulic shaker excitation for building and bridge, among others.

On-site testing can also be conducted using output measurement only. Free decay response (FDR) can be measured directly when the structure is subjected to transient, e.g. step function, excitation. In many occasions, ambient, or natural, excitation is preferred in field-testing. Actually, ambient testing has lots of advantages compared to lab testing. Ambient testing is cheap and fast, no elaborate excitation equipment required, no boundary condition simulation needed; Dynamic characteristics of the whole system, instead of component, can be obtained directly; In the operational condition the structure is subjected to real external loading, which is usually differ significantly from the excitation in lab testing;

In the output only cases, stochastic responses subjected to ambient excitation are measured. Auto and Cross Correlation Function (A-COR and C-COR) can be estimated via direct FFT computation. It should be noted that COR estimation via FFT computation is much faster than directly calculated for measured time series, but will usually results bias error. Zero-padding technique is used to obtained unbiased estimates [2]. Another technique is to estimate Random Decrement (RDD) signature. RDD was originally explained as a free decay response (FDR) [3], and a theoretical explanation is then developed and shows that RDD is actually a correlation function [4]. A diversity of techniques to calculate RDD has been proposed [5].

A generalized Time Response Function (TRF) is defined in this paper to cover FDR, A-COR, C-COR, as well as RDD signature. The TRF has different representation and can be estimated in different ways, but can be described via unified analytical formulation as the summation of exponentially decayed sinusoids.

3. Three Damping Estimation Approaches

3.1 Time Domain Approach via Data Correlation (DC)

The classical logarithmic decrement (LogDec) method is commonly used to give a quick estimate of the damping ratio from a Time Response Function (TRF). The main drawback of the LogDec is its sensitivity to the noise pollution.

Hilbert Transform (HT), developed in 1980's, can also be adopted to modal frequency and damping estimation effectively [6]. HT is defined as following convolution

$$x^H(t) = H\{x(t)\} = x(t) * \frac{1}{\pi t} = \frac{1}{\pi}\int_{-\infty}^{\infty} x(t)\frac{1}{t-\tau}\,d\tau \qquad (1)$$

An analytic signal of $x_a(t)$ is defied as

$$x_a(t) = x(t) + jx^H(t) \qquad (2)$$

A complex signal $x_c(t)$ is assumed to approximate the analytic one,

$$x_a(t) \cong x_c(t) = A(t)e^{j\varphi(t)} \qquad (3)$$

Where the modulus $A(t)$ represents the instantaneous envelope, while the argument $\varphi(t)$ is the instantaneous phase and given by

$$A(t) = \left((x(t))^2 + \left(x^H(t)\right)^2\right)^{1/2},\ \varphi(t) = \tan^{-1}\left(x^H(t)/x(t)\right) \qquad (4)$$

When $e^{-\zeta_r \omega_r t}$ is a slowly varying function, the envelop in the semi-log plane is a straight line,

$$\ln(A_r e^{-\zeta_r \omega t}) = -\zeta_r \omega_r t + \ln(A_r) \qquad (5)$$

The slope of which is the decay rate. In the similar way, the modal frequency can be obtained as the slope of the instantaneous phase line.

In the HT method, only one data block with sufficient points is required to obtain a good estimate of damping ratio, even noise is presented. Linear regression averages out zero-mean stationary noise.

It has been shown [7] that data correlation, as a correlation filtering, is very powerful in noise reduction. When enough TRF data points are available, their correlation function can be computed and used as new TRFs for further parameter estimation. The zero-mean uncorrelated noise in the signal will then be “filtered” out very effectively. More accurate results can be obtained by data correlation.

Theoretically, LogDec and HT related methods, including their data correlation, can deal with only single degree-of-freedom (S-DOF) system. In practice, band pass filter can be employed to isolate the single mode as a S-DOF system. However, filter would bring distortion of the signal and causes bias error. Even though, it still difficulty to deal with closely spaced modes.

3.2 Time–Frequency Domain Approach via Wavelet Transform (WT)

Wavelet Transform (WT) of a signal is a time-scale decomposition obtained by dilating and translating a selected basic wavelet function. The continuous wavelet transform (CWT) is defined as follows,

$$w_x(a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) g^*(\frac{t-b}{a}) dt \qquad (6)$$

Where *b* is the translation parameter indicating the locality in time domain, *a* is a dilation or scale parameter localizing the signal in frequency domain, *g*(*t*) is an analyzing function called basic wavelet, $g^*(t)$ is its complex conjugate. The possibility of time-frequency localization arises from the basic wavelet being a window function, which decays very fast like a "small wave",

$$\int_{-\infty}^{\infty} g(t) dt = 0 \qquad (7)$$

CWT of a signal actually is a convolution of the signal and the selected wavelet function, and can be calculated in frequency domain via FFT.

$$w_x(a,b) = \sqrt{a} \int_{-\infty}^{\infty} X(f) G^*_{a,b}(af) e^{2\pi fb} df \qquad (8)$$

One of the most widely used functions for structural parameter identification is the well-known Morlet wavelet

$$g(t) = e^{-t^2/2} e^{j\omega_0 t} \qquad (9)$$

Its spectrum shows excellent frequency locality due to "sliding window" resulted from frequency domain transform of the wavelet (Figure 1). For a exponentially decayed time response function *x*(*t*), its WT with Morlet wavelet can be derived as

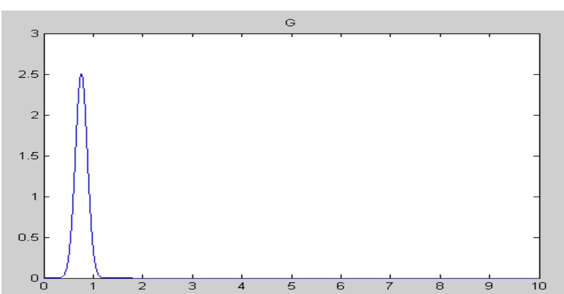

Fig. 1 The spectrum of Morlet wavelet acts as "sliding window"

$$w_x(a,b) = \sqrt{2\pi a} A e^{-\sigma_r b} e^{-2\pi^2 (af - f_0)^2} e^{j2\pi f_{dr} b} \qquad (10)$$

When choosing dilation parameter as $a=f_0/f$ and logarithm is applied, the modulus of the wavelet transform can be written as

$$\ln(|w_x(a,b)|) = -\zeta_r \omega_r b + \ln(A G^*_{a,b}(af_r)) \qquad (11)$$

Thus the damping ratio can be estimated from the slope of the straight line of the wavelet modulus cross-section, for given value of dilation *a*, plotted in a semi-logarithmic plane. The rest procedure of damping ration estimation is similar to the envelope analysis used in HT method (See Figure 2).

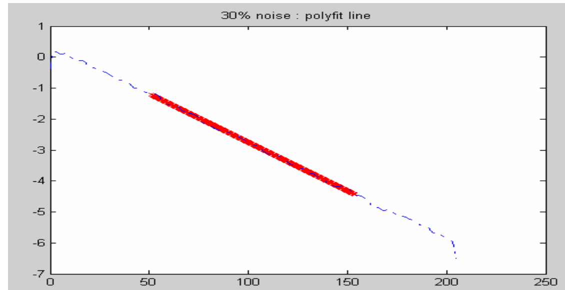

Fig. 2 Robust decay rate estimation obtained from wavelet modulus cross-section

The above formulation is derived from complex-valued analytic signal. However, it can be extended easily to the exponentially decayed TRF. CWT has been shown very effective in modal parameter estimation [8]-[10]. Numerical simulation of WT method has shown very encourage results, even with very noise TRF data. Moreover, WT possess two plausible features: one is that it works for multiple DOF systems; the other is the applicability with non-stationary measurement.

One of the major advantages of CWT is that multiple modes or close modes can be handled due to its "sliding window" effect, which can be observed from frequency domain transform of the continuous wavelet. However, very closely spaced mode cannot be separated from by CWT.

3.3 Spatial Domain Approach via Frequency Domain Decomposition (FDD)

Heavily modal coupling is often encountered with complicate engineering structures. Symmetrical and anti-symmetrical modes of a structure are often closely spaced. Axially symmetrical structure could possess modes with almost identical modal frequencies. Large complicated structures, e.g. spacecrafts and long-span flexible bridges, have dense modes in the frequency range of interest. The TD approach with data correlation and even Time-Frequency domain approach via wavelet transform will have difficulty in dealing with closely spaced or repeated modes. A new spatial domain approach based on Frequency Domain Decomposition (FDD) has been developed to resolve the difficulties [11].

FDD is also a two-stage non-parametric modal estimation approach. In the first stage, auto and cross Power Spectrum Densities (PSDs) are estimated based on random response data measured from spatially distributed sensors. Singular Value Decomposition (SVD) is conducted as the first step of the second stage,

$$G_y(f) = \Phi \Sigma(f) \Phi^T \qquad (12)$$

When making all measurement coordinates as references, SVD is equivalent to Eigenvalue Decomposition (EVD). It has been shown [11] that the singular value or eigenvalue of the PSD, as function of frequency, can be represented as a single mode

$$\Sigma(f) = diag.\left[\sigma_r(f)\right], \sigma_r(f) = \frac{d_r}{j2\pi f - s_r} \tag{13}$$

Where s_r and d_r are r-th complex modal frequency and residue, respectively. Actually the above result can also be explained via modal decomposition of PSD,

$$G_y(f) = \Phi G_q(f) \Phi^T \tag{14}$$

where $G_q(f)$ is in modal coordinate. PSD can be expressed as Fourier transform of the covariance, or correlation function,

$$\begin{aligned} R_y(\tau) &= E\{y(t+\tau)y(t)^T\} \\ &= E\{\Phi q(t+\tau)q(t)^T \Phi^T\} \\ &= \Phi R_q \Phi^T \end{aligned} \tag{15}$$

Where $y(t) = \Phi q(t)$,R_q is the correlation function matrix in modal space, and therefore it's Fourier transform $G_q(f)$ should be diagonal.

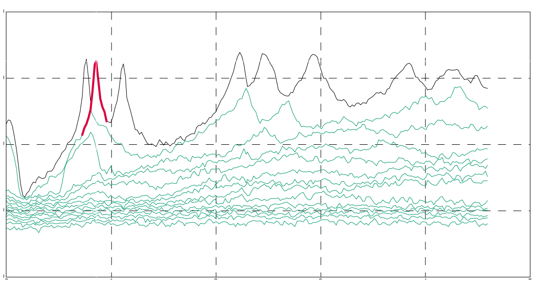

Fig. 3a SV Plot from 15-story building, The "Bell" shows the enhanced PSD data of 2nd mode.

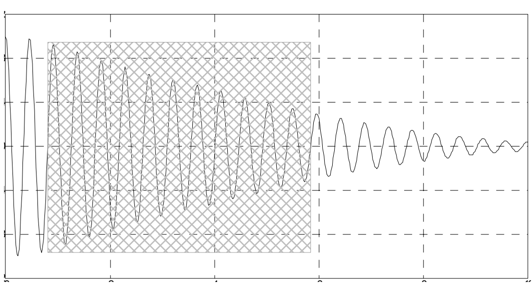

Fig. 3b Time response function of the "Bell", very close to the TRF of the 2nd mode

The first singular value (SV) spectrum, as a function of frequency, is an enhanced PSD (Figure 3a). The data in the vicinity of the peaks of the first SV plot, their corresponding MAC value is close to 1, represent the modal responses. The inverse FFT of these data, which look like a "bell", is a very close approximation of TRF of S-DOF system, which can be easily observed in Figure 3b [12]. Therefore, the previously introduced damping estimation methods can be adopted as the third step of the second stage.

It has been shown by theoretical analysis, numerical simulation and practical application that FDD can handle not only closely spaced modes, but also repeated modes [11],[13]. Another significant advantage of this approach is that FDD is very robust to the noise pollution, for SVD has a strong feature of separating the noisy data into "signal space" and "noise space".

4. Numerical Simulations and Comparisons of the Three Approaches

In order to show the performance of the three damping estimation approaches, numerical simulations of a 15-story office building [4] (Figure 4a) and a communication transmission tower [5] (Figure 4b) are conducted. A set of exponentially decayed response data with different amount of white noise added were created to simulate the TRFs of the first bending mode (f_1=0.74 Hz) of the 15-story office building [12] are utilized for the first numerical simulation.

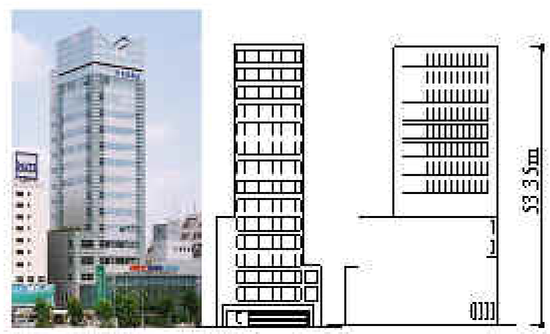

Fig. 4a A 15-story Office Building

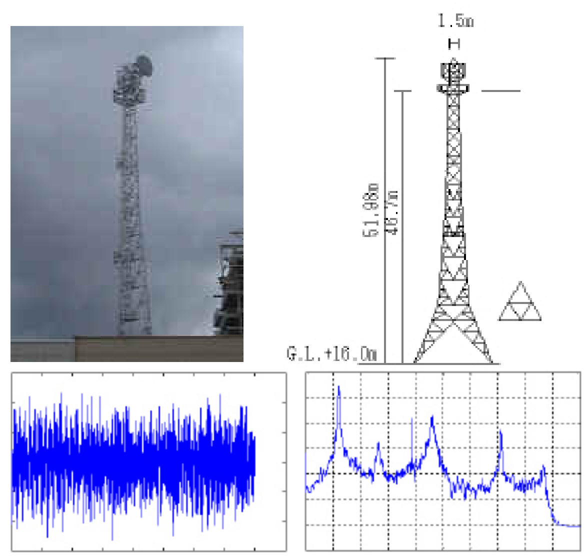

Fig. 4b A Transmission Tower

Figure 5 shows the linear regression of LogDec with data correlation using simulation data polluted by 50% noise. Figure 6 depicts the result estimated via HT with data correlation using same noisy data. It is seen that pretty

accurate results have been obtained. As comparisons, Figures 7 and 8 give the corresponding results via LogDec and HT techniques without data correlation. Only 10% and 30% noise were added, respectively, in these cases.
Table 1 summarized the results of LogDec, HT and their data correlation (R-LogDec, R-HT), as well as WT methods using different level of noise polluted TRF data

Table 2 gives the results of the second numerical simulation via CWT, which has three modes with modal frequencies of 1.20, 2.65 and 3.85 Hz. Only CWT method was employed for damping estimation in this M-DOF case. It is seen that very accurate damping can be obtained with very noisy data in multi-mode case with WT method.

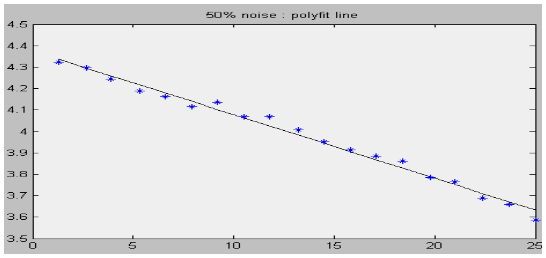

Fig. 5 Damping estimation via Data Correlation of LogDec, 50% noise.

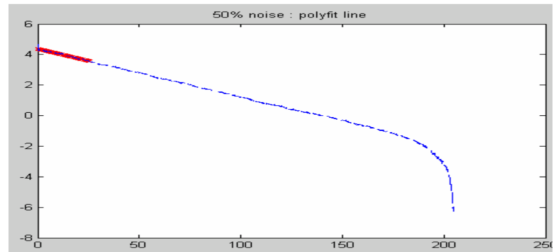

Fig. 6 Damping estimation via data correlation of HT, 50% noise

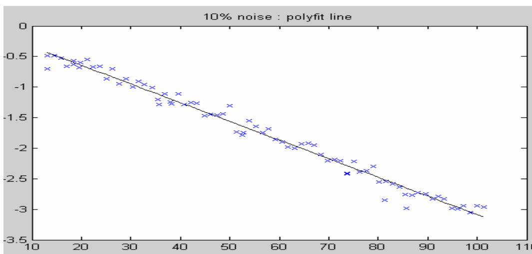

Fig. 7 Damping estimation of LogDec without data correlation, 10% noise

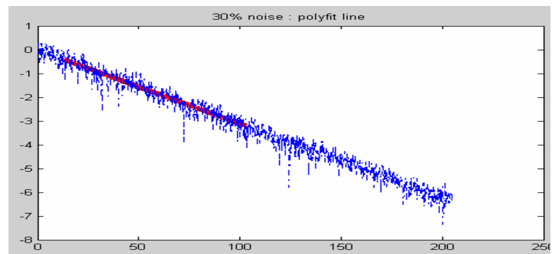

Fig. 8 Damping estimation of HT without data correlation, 30% noise

Table 1 Numerical Simulation Results of the first mode of a tall building

		Target	Damping Estimation Results with Noisy TRF data					
			5%	10%	30%	50%	70%	100%
X-LOG	Damp (%)	0.6500	0.6469	0.6098	0.4244	0.3287	0.2804	0.2414
	Error. (%)	/	-0.48	-6.2	-34	-49	-57	-63
X-HT	Damp (%)	0.6500	0.6501	0.6518	0.6511	0.5921	0.4862	0.4003
	Error. (%)	/	0.015	0.28	0.17	-8.9	-25	-38
R-LOG	Damp (%)	0.6500	0.6473	0.6470	0.6525	0.6490	0.6450	0.6719
	Error. (%)	/	-0.18	-0.46	0.38	-0.15	-0.77	3.4
R-HT	Damp (%)	0.6500	0.6496	0.6534	0.6529	0.6355	0.6589	0.6307
	Error. (%)	/	-0.061	0.52	0.45	-2.2	1.4	-3.0
X-WT	Damp (%)	0.6500	0.6500	0.6500	0.6592	0.6463	0.6545	0.6656
	Error. (%)	/	0.0	0.0	1.4	-0.57	0.69	2.4

Table 2 Results of Damping Estimation for a Transmission Tower

		Target	Damping Estimation Results with Noisy TRF data					
			5%	10%	30%	50%	70%	100%
1st Mode	Damp (%)	1.0000	0.9531	0.9576	0.9715	0.9612	0.9632	0.9519
	Error. (%)	/	-4.7	-4.2	-2.9	-3.9	-3.7	-4.8
2nd Mode	Damp (%)	0.8000	0.7832	0.7797	0.7644	0.7260	0.7525	0.6794
	Error. (%)	/	-2.1	-2.5	-4.4	-9.3	-5.9	-15
3rd Mode	Damp (%)	0.5000	0.4886	0.5002	0.4866	0.4407	0.4716	0.4622
	Error. (%)	/	-2.3	0.048	-2.7	-12	-5.7	-7.6

5. Concluding Remarks

1. Time domain, time-frequency domain and spatial domain approaches have been introduced for damping estimation of engineering structures using ambient response data. Computer simulations based on ambient response measurements and modal identification of a 15-story office building and a transmission tower have been conducted to evaluate the performance and compare their capability in dealing with noisy data and closely space modes.

2. Logarithmic Decrement (LogDec) and Hilbert Transform (HT) techniques, and their Data Correlation (DC) versions work only for S-DOF system in theory. Making use of a suitable band pass filter, pretty accurate results can be obtained for lightly or medium coupled modes. With data correlation, very noisy data can be dealt with. However, enough data points are required when data correlation is adopted for only first, e.g., 10 % to 15 % of the correlation data can be used to accurately estimate damping.

3. One of the attractive features of WT method is multiple DOF system can be handled directly owing to its time-scale decomposition. Thanks to the "sliding window" of the CWT, the time-frequency domain approach can handle very noisy measurements. Another important advantage of the WT approach is that the stationarity assumption for the ambient response, as stochastic process, is no longer necessary. It has significant importance to the ambient structural system identification.

4. Frequency Domain Decomposition is a spatial domain approach. It is simple but powerful in modal parameter estimation, and has the ability to deal with closely spaced or even repeated modes. Due to the employment of the Singular Value Decomposition (SVD), this spatial domain approach is very robust to noise pollution in the measured time response data.

5. However, there are many challenges in damping estimation to be addressed. The accuracy of the damping estimation for engineering structures is one of the most important issues in response prediction and vibration control. Basically all the methods mentioned above are two-stage approach. Estimation of correlation function (COR) or random decrement (RDD) signature is the first stage. There are many error sources, bias and variance in nature, in time response Function (TRF) estimation. For example, whenever FFT is adopted to calculate TRF, leakage will occur which can cause significant bias error in the second stage of damping estimation. Variance error in TRF, which increases when data length is limited, will also result in bias error in damping estimation.

6. All the damping estimation techniques discussed in this paper can be classified as non-parametric methods. An array of parametric modal parameter estimation procedures has been developed the last 30 years. Most time domain (TD) parametric modal identification can work for not only input/output measurement, but also output data only. Therefore, they can be employed for damping estimation of engineering structures with ambient response measurements. One of the major challenges for parametric modal identification is, again, the accuracy of the damping estimation. Bias and variance errors in the first stage will cause significant error when applying parametric damping estimation techniques, which are mostly based on least squares estimation. Another serious issue for all TD parametric identification approaches is how to distinguish structural modes from "noise" (or fictitious) modes, which are introduced to accommodate measurement noise, leakage, out-of-band modes and non-linearity, etc. and play an important role in TD modal identification. However, the problem of how to differentiate true structural modes from spurious ones is still remaining to be solved, and noise mode problem can cause severe error in damping estimation.

7. One more significant challenge in damping estimation comes from theoretical aspect. When the damping in the structural system is assumed to be proportional, normal mode is resulted. Therefore, M-DOF system can be decomposed into S-DOF system in modal space. Modal frequencies, as undamped natural frequencies of the linear structure, can be obtained from real part (damping coefficients) and imaginary part (as damped natural frequencies) of the eigenvalues or poles of the system. However, for real engineering structures, damping is hardly to be proportional. Modal frequency and modal damping calculated from complex eigenvalues are NO LONGER equal to the undamped natural frequency and damping ration as in proportional damping case! Therefore, we need either an appropriated measure for non-proportional damping, or a new description of the damping property.

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REFERENCES

[1]. Tompson, W.T. *Theory of Vibration with application*, 3^{rd} Ed. Prentice Hall, Englewood Cliff, New Jersey, 1993

[2]. Bendat, J. & Piersol, A., Random Data, Analysis and Measurement Procedures, John 'Wiley & Son, New York, 1986

[3]. Ibrahim, S. R. "Random Decrement Technique for Modal Identification of Structures," *Journal of Spacecraft and Rockets*, Vol. 14, 1977, pp. 696-700

[4]. Vandiver, J. K. et al., A Mathematical Basis for Random Decrement Vibration Signature Analysis Technique, *ASME J. of Mechanical Design*, Vo.104, April 1982

[5]. Asmussen, J.C., Modal Analysis based on the Random Decrement Technique, Ph.D. Thesis, Aalborg University, Denmark, 1997

[6]. Agneni, A. and Balis Crema, L. Analytic Signal in the Damping Coefficient Estimation, *Proc. Of the Int'l Conf. On Spacecraft and Mechanical Testing*, Noordwijik, The Netherlands, 1988

[7]. Zhang, L.-M et al., An Improved Time Domain Polyreference Modal Identification Method, *Mechanical System & Signal Processing*, 1 (3), 1987

[8]. Russene,M et. al. Natural Frequency and Damping Identification using Wavelet Transform, *Mechanical System& Signal Processing,* 11(2), 1997

[9]. Lamarque,C.H. et. al. Damping Identification in M-DOF system via Wavelet-Logarithmic Decrement-I: Theory, *J. of Sound and Vibrat*ion, 235(3), 2000

[10]. Zhang, L.-M. and Tamura, Y., Damping Estimation of Engineering Structures, Advances and Challenges, Seminar on Identification of Dynamic Characteristics of Structural System, Wind Engineering Center, Tokyo Institute of Polytechnics, Japan, Oct. 12, 12002

[11]. Brincker, R., Zhang, L.-M. and Anderson, P., Modal Identification from Ambient Response using Frequency Domain Decomposition, *Proc. of the 18th IMAC,* San Antonio, TX, USA, Feb. 7-10, 2000

[12]. Brincker, R., Ventura, C.E. and Anderson, P. Damping Estimation by Frequency Domain Decomposition, *Proc. of the 19th IMAC*, 698-703, Feb. 2001

[13]. Tamura, Y., Zhang, L.-M. and Yoshida, A. Ambient Vibration Testing and Modal Identification of an Office Building, Proc. Of the IMAC XX, Los Angeles, CA, USA, Feb. 2002