

An Overview of Operational Modal Analysis: Major Development and Issues

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Abstract

An overview of the major development of operational modal identification is presented. It includes four time domain approaches, i.e. NExT-type, ARMAV model-based, stochastic realization-based, stochastic subspace-based methods, and two frequency domain approaches, i.e. FDD-type and output-only LSCF-type methods. The internal relation with traditional modal identification using input/output measurements for EMA, as well as with classical and modern system identification are revealed. The major issues in OMA are also summarized, which cover full or partial references, multiple sensor setups, nonstationary excitation, data-driven vs covariance-driven SSI, structural mode sorting, bias or unbiased modal parameter estimation, as well as mode shape scaling.

1 Introduction

Modal analysis has been widely applied in vibration trouble shooting, structural dynamics modification, analytical model updating, optimal dynamic design, vibration control, as well as vibration-based structural health monitoring in aerospace, mechanical and civil engineering.

Traditional experimental modal analysis (EMA) makes use of input (excitation) and output (response) measurements to estimate modal parameters, consisting of modal frequencies, damping ratios, mode shapes and modal participation factors. EMA has obtained substantial progress in the last three decades. Numerous modal identification algorithms, from Single-Input/Single-Output (SISO), Single-Input/Multi-Output (SIMO) to Multi-Input/Multi-Output (MIMO) techniques in Time Domain (TD), Frequency Domain (FD) and Spatial Domain (SD), have been developed.

However, traditional EMA has some limitations, such as:

- 1). In traditional EMA, artificial excitation is normally conducted in order to measure Frequency Response Functions (FRFs), or Impulse Response Functions (IRFs). FRF or IRF would be very difficult or even impossible to be measured in the field test and/or for large structures;
- 2). Traditional EMA is normally conducted in the lab environment. However, in many industrial applications, the real operation condition may differ significantly from those applied in the lab testing;
- 3). Component, instead of complete system, is often tested in the lab environment, and boundary condition should be reasonably simulated.

Since early 1990's, operational modal analysis (OMA) has drawn great attention in civil engineering community with applications for off-shore platforms, buildings, towers, bridges, etc. OMA, also named as ambient, natural-excitation or output-only modal analysis, utilizes only response measurements of the structures in operational condition subjected to ambient or natural excitation to identify modal characteristics. OMA is also very attractive for aerospace and mechanical engineering due to many advantages, such as: (1) OMA is cheap and fast to conduct, no elaborate excitation equipment and boundary condition simulation are needed. Traditional modal testing is reduced to be response measurement; (2) Dynamic characteristics of the complete system,

instead of component, at much more representative working points can be obtained; (3) The model characteristics under real loading will be linearized due to broad band random excitation; (4) All or part of measurement coordinates can be used as references. Therefore, the identification algorithm used for OMA must be MIMO-type. As a consequence, the closed-spaced or even repeated modes can easily be handled, and hence suitable for real world complex structures; (5) Operational modal identification with output-only measurements can be utilized not only for dynamic design, structural control, but vibration-based health monitoring and damage detection of the structures.

In this paper, an overview of the development of operational modal identification is presented and major issues in OMA are discussed.

1. Major Developments of OMA

2.1 OMA in Time Domain, NExT-type procedures

In early 1990s, Natural Excitation Technique (NExT) was proposed for modal identification from output-only measurements in the case of natural excitation [1]. NExT actually is an idea or principle that suggests using correlation function (COR) of the random response of the structure subjected to natural excitation. NExT has shown that the COR can be expressed as a summation of decaying sinusoids. Each decaying sinusoid has a damped natural frequency, damping ratio and mode shape coefficient that is identical to the one of the corresponding structural mode. COR can therefore be employed as impulse response function (IRF) to estimate modal parameters. Hence, major multi-input/multi-output ((MIMO) TD modal identification procedures developed in traditional EMA can be adopted for OMA.

There are three major TD MIMO algorithms have been widely utilized in EMA:

- ✧ Polyreference Complex Exponential (PRCE), developed in 1982 [2] as an MIMO extension of the SIMO Least Squares Complex Exponential (LSCE) [3];
- ✧ Eigensystem Realization Algorithm (ERA), adopted from system realization theory in linear system analysis, and applied for modal identification in 1984 [4]
- ✧ Extended Ibrahim TD (EITD), an MIMO version of ITD [5], implemented in 1985 [6]

To reduce the influence of noise contamination in the IRF data, an improved PRCE called Improved Polyreference technique (IPCE), which makes use of correlation of the TRF data, was developed in 1987[7]. Data correlation version of ERA, as ERA/DC [8], followed in 1988.

Actually, Impulse Response Function (IRF), Free Decay Response (FDR), Correlation Function (COR) as well as Random Decrement signature (RDD) can all be expressed as modal superposition or modal decomposition, i.e. a summation of exponentially decayed sinusoids. A Time Response Function (TRF) is proposed to represent these TD signature or features [9]. The NExT-type of operational modal identification procedures can also be called as two-stage TD modal identification, as aforementioned traditional TD MIMO: estimation of the time response function (TRF) as the first stage, and extraction modal parameters from TRF data as the second stage.

The time response function (TRF) can be estimated by the following techniques:

- ✧ Inverse FFT of the FRF estimation to obtain IRF for time domain EMA;
- ✧ Free decay response (FDR) can directly be measured either from transient excitation or sudden termination of broad band random excitation;
- ✧ Correlation function (COR) can either be estimated directly from stochastic response via correlogram, or from Power Spectrum Density (PSD) via inverse FFT via periogram;
- ✧ Random decrement signature (RDD) was proposed as free decay response in the beginning [10], [11], and then proved to be correlation function (COR) of the response, and can be computed through many ways from random response data [12], [13].

It should be noted that the NExT-type modal identification procedures adopted from EMA are all developed in the deterministic framework. However, in OMA the data utilized for modal parameter estimation are random response, which is stochastic process. Although the major traditional modal identification algorithms, such as PRCE, EITD, ERA, etc., developed for EMA, can be employed for OMA, the difference in theoretical background and basic formulation should be notified.

PRCE and EITD procedures are based on the modal decomposition of impulse response function (IRF), derived from first order equation of linear time-invariant (LTI) system in modal coordinates:

$$H(t) = \Phi e^{\bar{\Lambda}t} \Gamma^T \text{ or } H_k = \Phi Z^k \Gamma^T, Z = e^{\bar{\Lambda}\Delta t}, t_k = k\Delta t$$

The formula is derived from differential equations of linear time-invariant system (LTI):

$$\begin{aligned} M\ddot{\xi}(t) + D\dot{\xi}(t) + K\xi(t) &= bf(t) \text{ or } \dot{x}(t) = A_c x(t) + B_c u(t), & x &= \begin{Bmatrix} \xi \\ \dot{\xi} \end{Bmatrix}, & A_c &= \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} \\ y(t) &= c\xi(t) & y(t) &= Cx(t) \end{aligned}$$

In the modal decomposition formula, $\bar{\Lambda}$ is the eigenvalue matrix of A_c , from which modal frequencies and damping ratios can be calculated. Φ , Γ are mode shape matrix and modal participation factor matrix, respectively. There are three steps for modal identification: (1) extraction of Φ and Z matrices, (2) calculation of modal frequencies and damping ratios from Z matrix, (3) computation of modal participation factor matrix Γ .

In the OMA case, modal decomposition of COR matrix, instead of IRF matrix, is utilized as basic formulation for modal identification:

$$R_{yy}(t) = \Phi e^{\bar{\Lambda}t} \Gamma_R^T, \Gamma_R^T = \alpha \int_0^\infty e^{\bar{\Lambda}\sigma} \Gamma^T \Gamma e^{\bar{\Lambda}\sigma} d\sigma \cdot \Phi^T$$

It is seen that only first two steps are required for estimation of modal frequencies and damping ratios and mode shapes. No modal participation factor matrix can be obtained due to missing of input information.

On the other hand, traditional ERA is based on the system decomposition of IRF matrices:

$$H(t) = C e^{A_c t} B_c$$

Modal frequencies and damping ratios can be computed from the eigenvalues of the system matrix A_c , mode shape and modal participation factor matrices can be calculated from the eigenvector matrix Φ plus output matrix c and input inference matrices b .

In the OMA case, instead of IRF, COR matrix is utilized. The COR matrix can be derived from stochastic state-space representation as

$$R_{yy}(t) = C e^{A_c t} B_R, B_R = \alpha \int_0^\infty e^{A_c \sigma} B_c B_c^T e^{A_c \sigma} d\sigma \cdot C^T$$

Obviously, system matrices A_c and C can be estimated as before via ERA algorithm. However, as an OMA procedure, no modal participation factor matrix can be identified.

2.2 OMA in TD, ARMA-type procedures

A general auto-regression moving average (ARMA) model can also be employed for operational modal identification. Corresponding to multiple natural excitations, multi-dimensional ARMA model, i.e. Vector ARMA or ARMAV model should be applied.

The most important traditional system identification techniques are the Prediction-Error Method (PEM) [14]. A number of algorithms in the PEM framework has been proposed. These algorithms identify the parameters of a model by minimizing the prediction errors. The application of PEM to estimate an ARMA model results in a highly nonlinear optimization problem. PEM-ARMAV methods have applied to ambient modal identification in the middle of 1990's [15]. Modal parameters can be computed from the ARMAV model by the coefficient matrices of the AR

polynomials. Two major drawbacks are inherent for PEM-ARMAV type OMA procedures: computational intensive and requirement of initial “guess” for the parameters to be identified. It makes ARMAV-type procedures rather difficult to apply, especially for large dimension structures.

A Linear Multi-Stage (LMS) ARMAV method for effective OMA in the presence of noise is proposed recently [16]. The LMS-ARMAV method overcomes some of the difficulties that have rendered ARMAV identification in use for mechanical structures.

System identification based on ARMA model is a non-linear process and should be implemented in iteration way. The non-linearity is caused by the MA parameters. However, for the modal identification, only modal parameters are interested, that means only the coefficient matrices of the AR polynomial are required. For this purpose, another important approach in the classical system identification, i.e. Instrument Variable Method (IVM) can be applied. Making use of “past” output data as “instrument variable”, which is assumed uncorrelated to the input (white noise) residuals, the original ARMA model turns to be AR model and its coefficient matrices are nothing but covariance (COV) matrices of the output data. It should be noted that covariance function (COV) is equal to correlation function (COR) for the zero-mean random process. Although derived in different way, the final equations from the ARMAV model-based IVM are exactly the same as from NExT-type PRCE or EITD, where COR is utilized for modal identification. The dimension of the AR coefficient matrices is equal to the number of inputs or references for PRCE, and number of output for EITD. Modal parameters can then be calculated by the eigenvalue decomposition of the companion matrix of the AR polynomial [17]~ [19]. Due to noise contamination, actual model order should be defined much higher in practice.

It is worth noticing that PEM-ARMAV is one-stage, or “data-driven” approach, and IVM-ARMAV methods, or MExT-type PRCE and EITD, belong to two-stage, or “covariance-driven” approach.

2.3 OMA in Time Domain, Stochastic Realization-based Procedures

The ARMAV model-based methods are primarily used with so-called black-box model structures. The use of such models is quite cumbersome in multivariable case. For large scale system, a state-space model is preferred. As mentioned before that NExT-type ERA for OMA is actually one of the stochastic system realization approaches based on state-space model.

System realization, i.e. recovering or identification of system matrices, was developed way back to 1960’s. A classical contribution in deterministic system realization by Ho & Kalman was published in 1966, where a scheme for recovering the system matrices from impulse response function (IRF) is outlined [20]. Refinements of the scheme are reported in 1974 and 1978, respectively, introducing singular value decomposition (SVD) as a tool to reduce the noise inference in the IRF measurements [21], [22]. The SVD-based system realization was firstly adapted for modal identification and named as Eigensystem Realization (ERA) in 1984.

Almost in parallel, stochastic system realization was developed in 1970’s [23], based on discrete-time stochastic state-space equation, and applied to modal identification in middle of 1980’s [24]. The key feature of the stochastic system realization is the system decomposition of the COV matrix instead of IRF matrix in deterministic system realization. It leads to factorization of Hankel covariance matrix.

Since eigenvalues of the system are calculated via SVD of IRF matrices (for EMA), or covariance matrix (for OMA), the system realization-based approaches can also be taken as subspace system identification method. In the modal community, stochastic realization based procedures are often called as Covariance-driven Stochastic Subspace Identification (SSI) methods.

There are three major methods for implementation of stochastic realization-based or covariance-driven SSI procedures: (1) the Principal Component (PC) method, (2) The Canonical Correlation, or Canonical Variant Analysis (CVA) method and (3) The Un-weighted Principal Component (UPC) method. In PC method, Hankel covariance matrix is directly utilized for SVD. In CVA and UPC methods, weighted Hankel covariance matrix is adopted via different weighting [25].

It can be shown that UPC method performs balanced model reduction on minimum-phase model corresponding to the given covariance. Therefore, UPC is also called as Balanced Realization (BR). It is interesting to note that UPC method, not PC one, is a stochastic counterpart of the deterministic realization/identification algorithm, e.g. ERA.

The physical explanation of the three methods in stochastic system realization is different, i.e. to select partial states maximizing correlation w.r.t. output prediction (PC), or mutual information w.r.t. future response (CVA), or retaining maximum reconstruction (prediction) efficiency for the output (UPC). The principal components of the resulting Hankel matrices H_{PC} , H_{CVA} and H_{UPC} are, of course, all different. However, computer simulation, as well practical application, reveals that no significant accuracy difference has been observed for PC, CVA and UPC implementations of the stochastic realization-based OMA procedures.

The main procedure of the stochastic realization-based operational modal identification can be summarized as following four steps:

- 1). Estimation of covariance matrix from measure output data;
- 2). SVD of (weighted) Hankel covariance matrix to estimate observability and stochastic controllability matrices;
- 3). Computation of discrete-time stochastic system matrices via LS techniques;
- 4). Calculation of modal parameters from the system matrices.

As in the deterministic case, block Hankel matrix and its shifted version can be used to estimate the system matrix, which is corresponding to ERA. Instead of Hankel matrix shift, the other approach is to utilize observability matrix and its shift to estimate system matrices. There are few possible numerical solutions to estimate the system matrix, e.g. ordinary least squares (OLS), total least squares (TLS) and partial least squares (PLS) techniques, etc.

2.4 OMA in Time Domain, Stochastic Subspace-based Procedures

In 1990's, a new subspace-based state-space system identification (4SID) is developed in system and control engineering, which offers numerically reliable and effective state-space model for complex dynamic system directly from measured data [26]. No non-linear search is required and computational complexity is therefore dramatically reduced compared to PEM-ARMAV or LMS-ARMAV procedures. Stochastic Subspace Identification technique (SSI) has been followed making use of output-only measurements for a system subjected to stochastic excitation [27].

Since the state vector is not observable, in order to predict the system response, its prediction (or estimate) should be utilized. It can be proved that the optimal predictor/estimate of the state, in the Gaussian case, is the condition mean of the state given all previous output measurements. Its linear algebraic explanation is the orthogonal prediction of the state vector to the "past" output data vector. Making use of state predictor/estimate leads to Kalman filter for LTI system, and can be expressed by so-called innovation state-space equation model:

$$\hat{x}(k+1) = A\hat{x}(k) + Ke(k)$$

$$y(k) = C\hat{x}(k) + e(k)$$

Where the predicted state $\hat{x}(k)$ is called Kalman state, K is Kalman Filter Gain and $e(k)$ is the innovation, which is a zero-mean Gaussian white noise process. The major difference between two

state-space representations is that the state vector is substituted by its prediction, and that the two input/noise processes have been converted into one, i.e. innovation.

The main procedure of the SSI technique can be summarized as following four steps:

- 1) Computation of the projection of the row space of the “future” outputs on the row space of the “past” outputs via robust numerical technique such as QR decomposition;
- 2) Estimation of Kalman filter state via SVD of the above projection matrix;
- 3) Estimation of the discrete-time system matrices via LS techniques;
- 4) Calculation of modal parameters as before.

Similar to the stochastic realization-based approach, the stochastic subspace-based identification can be implemented in three methods depending on the choices of weighting matrices for projection matrix: (1) the Un-weighted Principal Component (UPC) method, (2) the Principal Component (PC) method and (3) the Canonical Variant Analysis (CVA) method.

The advantages of SSI are that (1) it makes direct use of stochastic response data without estimation of covariance as first stage; (2) it cannot only be employed for white noise excitation, but also for color noise.

SSI has soon been adopted for operational modal identification. Since SSI makes direct use of stochastic response data to identify modal parameters, it is also called data-driven SSI. The data-driven SSI is belongs to one-stage modal identification category.

2.5 OMA in Frequency Domain, FDD-type Procedures

Frequency domain OMA methods are simply based on the formula of input and output power spectrum density (PSD) relationship for stochastic process: [28].

$$G_{yy}(j\omega) = H(j\omega)^* G_{xx}(j\omega) H(j\omega)^T$$

Where $G_{xx}(j\omega)$, $G_{yy}(j\omega)$ are input and output PSD matrix, respectively, $H(j\omega)$ is the FRF matrix, which can be expressed as partial fractions form via poles λ_r and residues R_r

$$H(j\omega) = \sum_{r=1}^N \left(\frac{R_r}{j\omega - \lambda_r} + \frac{R_r^*}{j\omega - \lambda_r^*} \right)$$

Where $R_r = \phi_r \gamma_r^T$, ϕ_r, γ_r are mode shape and modal participation vector, respectively. When all output measurements are taken as references, then $H(j\omega)$ are square matrix and $\gamma_r = \phi_r$. Assuming input is white noise process, i.e. $G_{xx}(j\omega)$ equals to constant, the modal decomposition of the output PSD matrix $G_{yy}(j\omega)$ can be derived as modal decomposition:

$$G_{yy}(j\omega) = \sum_{r=1}^N \left(\frac{A_r}{j\omega - \lambda_r} + \frac{A_r^H}{-j\omega - \lambda_r^*} + \frac{A_r^*}{j\omega - \lambda_r^*} + \frac{A_r^T}{-j\omega - \lambda_r} \right)$$

Where r^{th} pole $\lambda_r = -\sigma_r + j\omega_{dr}$, corresponding r^{th} residue $A_r \approx d_r \phi_r^* \phi_r^T$, $d_r = \gamma_r^H G_{xx} \gamma_r$, is a real scalar for white noise excitation. In the vicinity of a modal frequency the PSD can be approximated:

$$G_{yy}^T(j\omega) \approx \phi_r \frac{2d_r}{j\omega - \lambda_r} \phi_r^H = \alpha_r \phi_r \phi_r^H$$

$\omega \rightarrow \omega_r$

Classical FD approach is Peak Picking technique (PP). PP is based on the fact that modal frequencies directly obtained from the PSD plot at the peak, and mode shapes can be obtained as a column of the PSD matrix at the corresponding damped natural frequency.

The PP technique gives reasonable modal estimates if the modes are well separated [29]. The main advantages compared to the TD techniques are that it has no bother with computational modes and

is much faster and simpler to use. However, for a complex structure, PSD peak picking technique is inaccurate. The accuracy of modal frequency estimation is limited to the frequency resolution of the PSD spectrum, operational deflection shapes is obtained instead of real mode shapes, damping ratio estimation via half-power point is inaccurate or impossible. Moreover, PP technique is hardly applied for the structure with closely spaced modes, which is often encountered when dealing with real world complex structures.

The challenge is if we could develop a FD technique that has all the advantages but doesn't have the disadvantages. A new FD technique named as Frequency Domain Decomposition (FDD) was proposed in 2000 [30]. The key of the second stage is conducting SVD of output PSD, estimated at discrete frequencies $\omega = \omega_i$,

$$G_{yy}(j\omega_i) = U_i S_i U_i^H$$

When only r^{th} mode is dominate at the modal frequency ω_r , The PSD matrix approximates to a rank one matrix as

$$G_{yy}(j\omega_i) \underset{\omega_i \rightarrow \omega_r}{=} s_i u_{i1} u_{i1}^H$$

Compared to the previous PSD approximation formula, it is seen that the first singular vector at the r^{th} resonance is an estimate of the r^{th} mode shape. $\hat{\phi}_r = u_{r1}$ In the repeated mode case, the rank of PSD matrix will be equal to the number of multiplicity of the modes. Therefore, the SV function can suitably be utilized as modal indication function (MIF). Modal frequencies can be located by the peaks of the SV plots. From the corresponding singular vectors, mode shapes can be obtained. Since SVD has the ability of separating signal space from noise space, the modes can be indicated from SV plots, and closely spaced modes or even repeated modes can easily be detected.

The first generation of FDD can only estimate modal frequencies and mode shapes. The second generation of FDD, which is called as Enhanced FDD or EFDD has been followed for estimation of not only modal frequencies and mode shapes, but damping ratios [31]. To do so, the singular value data near the peak with corresponding singular vector having enough high MAC value are transferred back to time domain via inverse FFT, which is approximation of correlation function of the SDOF system. From the free decay function of the S-DOF system, the modal frequency and the damping ratio can then calculated by the logarithmic decrement (Logdec) technique.

Since only truncated data, i.e. the data near the peak of the SV plot, are used for the inverse FFT to calculate approximate correlation function of the corresponding S-DOF system. It may cause bias error in damping estimation. Moreover, when dealing with closely spaced modes, beat phenomena would be encountered, which can leads inaccurate estimation of damping ratio by Logdec technique.

The third generation of FDD, i.e. Frequency-Spatial Domain Decomposition (FSDD), has been developed recently [32]. FSDD makes use of the singular vector, computed via SVD of output PSD with spatial measurements, to enhance the PSD. In most cases the enhanced PSD in the vicinity of a mode can be approximated as S-DOF system, and therefore an S-DOF curve fitter can be adopted to estimate relevant modal frequency and damping ratio.

2.6 OMA in Frequency Domain, LSCF-type Procedures

In the traditional EMA, the major FD modal identification approach is based on parametric transfer function model represented by rational fraction polynomial (RFP). The SISO, SIMO and MIMO versions of RFP method were developed in 1980's based on least squares solution [33]~[35]. In classical system identification, maximum likelihood (ML) estimators were developed to deal with noisy measurements [36]. A MLFD method was proposed making use of FRF measurements for

modal identification in late 1990's [37]. MLFD is a typical non-linear estimator, and should be implemented as iteration process. A least-squares complex frequency-domain (LSCF) estimation method was introduced to find initial values for the iterative MLFD method [37]. It was found that these "initial values" yielded enough accurate modal parameters with much smaller computational effort. However, LSCF, which is based on common-denominator model or scalar matrix-fraction description, has two major shortcomings: (1) mode shapes and modal partition factor are difficult to obtain by reducing the residues to a rank-one matrix using the SVD; (2) the closely spaced poles will erroneously show up as a single pole. A polyreference version of the LSCF method, based on right matrix-fraction model, has been developed recently [38]. With p-LSCF, the above mentioned shortcomings can be eliminated.

The aforementioned FD EMA methods are all based on modal decomposition of FRF matrix. In parallel, these methods can be adopted for OMA based on modal decomposition of half PSD, computed from FFT of the COR with positive time lags via correlogram, instead of FRF matrix [39], [40]!

3. Major Issues in OMA

3.1 Full or Partial References

From NExT point of view, we can pick up any output measurements as the references for OMA via PRCE. EITD and ERA procedures as mentioned before. In stochastic realization-based OMA or covariance-driven SSI, as well as data-driven SSI, the formulations developed based on using all the output measurements as references. To reduce the dimension of the involved matrices, and therefore the computational efforts, only a subset of the outputs is required for references [41]. However, there is theoretical question to be answered, where the reference and real input point are not exactly the same.

3.2 Multiple sensor setups

In reality, only limited of the sensors and/or data acquisition channels are available in the field response measurements. Therefore, multiple sensor setups are evoked. However, to merge data from different setups will fail when response process is non-stationary for different records. A covariance matrix normalization technique has been developed to resolve this issue. It is interested to note that normalization has extra advantage of smoothing out the nonstationarity in the data.

3.3 Robust w.r.t. Nonstationary Excitation

The output data will be nonstationary due to nonstationarity of the natural excitation in operational condition. In both covariance-driven and data-driven SSI, the system matrices, and therefore modal parameters, are only determined from observability matrix, and the excitation only affects the stochastic controllability matrix through the cross covariance matrix between state and output vectors. When the excitation is nonstationary, so is the response data. Therefore the estimated covariance matrix with limited output data would not converge. However, it is proved that approximate factorization of the covariance matrix still hold and the SSI algorithm can be applied to output data due to nonstationary excitation.

3.4 Data-driven vs Covariance-driven SSI

There are clear similarities between covariance-driven and data-driven SSI. The first step of both approaches can be referred to as data reduction: the former via covariance estimation from Hankel data matrix and the latter via data vector projection by QR-factorization of the Hankel data matrix. Two approaches can not only be applied for data reduction but also data averaging/smoothing. The SVD utilized in two approaches play a similar role in computing system matrices. It can be shown

that by an appropriate weighting for the Hankel covariance matrix, covariance-driven algorithms can be fitted into the same framework of the data-driven SSI;

There are a few advantages of data-driven SSI over covariance-driven ones:

- 1) The data-driven approach is numerically more robust due to its square root algorithm compare the matrix squared up in covariance-driven case;
- 2) Validation tools can be developed for data-driven SSI;
- 3) A spectrum formulation is available based on identified innovation state-space model.
- 4) Modal decomposition of the total response can be made with data-driven SSI

3.5 Structural mode Sorting

Most TD OMA methods, i.e. IVM-ARMAV methods, including NExT-type of PRCE and EITD, stochastic realization-based and stochastic subspace-based identification methods, are all based on sound theoretical background. However, all the TD modal identification methods encounter a very serious and difficult problem in properly determination of model order and distinguishing structural modes from spurious or noise modes, which must be introduced to accommodate measurement noise, leakage, residue, non-linearity and un-modeled effects. The tools developed until now, e.g. modal indication functions and stability diagrams, etc., have limited effect. Over or under-determination of structural modes results in inaccurate estimation of the modal parameters of the true structural modes, especially mode shapes and damping ratios.

On the other hand, FD methods perform much better in structural mode determination. FDD techniques can almost eliminate spurious mode problem via modal indication function obtained from SVD. The p-LSCF technique usually results a clean stability diagram in wide frequency range.

3.6 Bias and unbiased modal parameter estimation

Most operational modal identification procedures, as their counterparts in EMA, can be reduced to solve a set of linear equations. Ordinary Least Squares (LS) solution is normally adopted. Therefore, bias error could occur due to noise effects, including measurement noise, leakage, residue, nonlinearity and un-modeled effects. Even latest developed FD p-LSCF method can not avoid this bias issue. Theoretically, bias error can be reduced via employing TLS and PLS, etc. instead of ordinary LS. However, engineering practice has shown that little improvement can be obtained with rather heavy computational penalty.

Time domain PEM and frequency domain MLE have the advantage of taking noise model into account, and therefore can better handle noisy measurements, and having the capability of extracting unbiased modal parameters with confidence level. The main drawback is computation intensive and need initial “guess” for iterative search.

3.7. Mode Shape Scaling

One of the critical issues is that OMA can not offer mode shape scaling due to lack of input information. If the identified modal model is going to be used for structural response simulation or for structural modification, then the scaling factors of the mode shapes must be known. Also in health monitoring applications where damage is to be identified, the scaling factors might be important. Some suggestion has been given in the literature for solving this problem [42]. However, the approach gives exact answers only when there is a full set of modes, and robustness for a truncated modal space has not been demonstrated. Recently a new approach based on repeated testing, where mass changes are introduced at the points and the response is measured, have been proposed [43]~[45]. This approach seems more appealing, since to scale a certain mode only that particular mode needs to be known.

4. CONCLUDING REMARKS

The major development of the operational modal identification methods for OMA is presented. It includes four time domain approaches, i.e. NExT-type, ARMAV model-based, stochastic realization-based and stochastic sub-space approaches, and two frequency domain approaches, i.e. FDD-type and output-only LSCF-type methods.

Major TD modal identification methods in traditional EMA, such as PRCE, EITD and ERA, can be directly adapted for OMA from NExT point of view. However, it should be aware that the theoretical basis is different. Stochastic framework should be considered in OMA, instead of deterministic framework for EMA. It is interested to notice that OMA version of PRCE and EITD can more properly derived from ARMAV model via classical instrument variable method. The OMA version of ERA is actually one of the stochastic system realization algorithms.

Operational modal identification methods can also be classified as two-stage and one-stage approaches. The two-stage approach estimates covariance/correlation function (TD) or power spectrum density (FD) as first stage, and then extract modal parameters from output COV/COR or PSD data. NExT-type procedures are typical two-stage approach. PEM-ARMAV methods are typical one-stage approach. Covariance-driven and data drive stochastic subspace identification (SSI) are typical two-stage and one stage method respectively;

It is interesting to notice that not only NExT-type procedures can clearly find their counterparts in traditional EMA. Most output-only version of system realization-based and subspace-based modal identification methods can be adopted from their input/output counterparts. Since modal decomposition formula for output PSD matrix, equivalent to the one for FRF matrix, can be derived. Moreover, when correlogram is utilized to estimate COR and half PSD is obtained by using only the COR with positive time lag, the modal decomposition formula for output PSD looks exactly the same as the one for FRF matrix, except modal participation factor is replaced by reference vector. Therefore, most of FD modal identification techniques developed for traditional EMA can easily be adapted for OMA.

The major issues in OMA are also summarized in the paper, which cover full or partial references, multiple sensor setups, nonstationary excitation and response, data-driven vs covariance-driven SSI, structural mode sorting, bias or unbiased modal parameter estimation, as well as Mode Shape Scaling;

Accuracy issue would be more significant when dealing with many measurements with very noisy data from complex structures in operational condition. Mode shape scaling issue needs to be further studied when OMA is intended to be applied for structural response simulation, for structural modification, and for structural health monitoring with vibration-based damage identification;

Although engineering applications have shown that favorable modal parameters can be extracted under the nonwhite or/and even nonstationary natural excitation, further theoretical proof based on system identification and stochastic process is still missing.

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