A Frequency-Spatial Domain Decomposition (FSDD) Technique for Operational Modal Analysis

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Abstract

The development and theoretical background of a new operational modal identification technique, named as Frequency-spatial Domain Decomposition (FSDD) is described in this paper. Three applications in civil engineering structures for typical purposes, i.e. a. large-span stadium roof for verifying finite-element model, a highway bridge for damage detection and a long-span cable-stayed bridge for structural health monitoring, are presented.

1. Introduction

Experimental modal analysis (EMA) has been widely used for trouble shooting, structural dynamics modification, analytical model updating, optimal dynamic design; passive & active vibration control, as well as vibration-based structural health monitoring in aerospace, mechanical and civil engineering. An array of single-input/single output (SISO), single-input/multi- output (SIMO) and multi-input/multi-output (MIMO) modal identification (MID) algorithms in time, frequency and spatial domain have been developed in the past three decades. However, traditional EMA has some limitations: e.g. (1) artificial excitation is normally conducted in order to measure frequency response function (FRF) or impulse response function (IRF),which are typically used as primary data for subsequent modal parameter extraction.. Unfortunately, FRF or IRF are very difficult, or even impossible, to measure in the field testing or for large structures; (2) In many industrial applications, the real operation conditions may differ significantly from those for lab testing with artificial excitation; (3) Component, instead of complete system, is tested in the lab environment, and boundary condition should be reasonably simulated.

Operational modal analysis (OMA) of mechanical systems subject to ambient or natural excitation under operational condition has recently drawn great attention in civil engineering. OMA is also very attractive for aerospace and mechanical engineering due to many advantages, such as: (1) OMA is cheap and fast to conduct, no elaborate excitation equipment and boundary condition simulation are needed. Traditional modal testing is therefore reduced to be response measurement; (2) Dynamic characteristics of the whole system, instead of component, at much more representative working points, can be obtained; (3) The model characteristics under real loading will be linearized due to broad band random excitation; (4) All or part of measurement coordinates can be used as references; the identification algorithm used for OMA must be MIMO-type. As a consequence, the closed-spaced or even repeated modes can easily be handled, and hence suitable for real world complex structures; (5) Operational modal identification with output-only measurements can be utilized not only for structural control, but also in-situ vibration-based health monitoring and damage detection of the structures.

In the 1992's a Natural Excitation Technique (NExT) was proposed [1]. NExT is based on the principle that Correlation Functions (COR) measured under natural, e.g. ambient or operational excitation, can be expressed as a sum of exponentially-decayed sinusoids. Modal parameters, i.e. natural frequency, damping ratio and mode shape coefficient of each decaying sinusoid are identical to the ones of the corresponding structural mode. According to this principle, major time domain (TD) MIMO MID algorithms such as PRCE [2], Extended ITD [3], ERA [4] and their extension [5], [6] which are successfully and widely employed for traditional EMA, can be adopted for operational MID by using COR instead of IRFs. The COR functions can be obtained via direct estimation, inverse Fourier Transform from PSD, or via Random Decrement technique from random response subjected to broadband natural excitation.

Many sophisticated TD operational modal identification methods have been proposed in the last 10 years based on classical and modern system identification theory. NExT-type PRCE and EITD found better theoretical explanation based on multi-dimensional, or vector autoregressive moving average model (ARMAV) via instrument-variable method (IVM). NExT-type ERA is nothing but an implementation of the stochastic realization-based methods [7]. A powerful tool named as Subspace State-space System Identification (4SID) method [8] was developed in 1990's, and adopted by modal community afterwards. Stochastic Subspace technique (SST) was then proposed for output-only measurements [9]. In contract to classical prediction-error method (PEM), which is a typical non-linear identification and therefore computational intensive, SST has many advantages.

However, all TD modal identification algorithms have a serious problem in model order determination. Noise (spurious) modes are always generated when extracting structural (physical) modes. These computational modes are even necessary to accommodate unwanted effects, such as measurement noise, leakage, residuals, non-linearity and un-modeled characteristics, etc. The computational modes fulfill an important role in that they permit more accurate modal estimation by supplying statistical DOF to absorb these effects. In the traditional modal identification for EMA, IRF can be obtained via inverse FFT of FRF, and might need less computational modes. For operational modal identification, which makes use of correlation function calculated from random response data, the problems with model order determination and structural modes distinguishing become much more significant. For differentiation between real and spurious modes, many modal validation techniques have been developed. An array of modal indicators was developed for the purpose. Graphical approach making use of stability diagram is more widely adopted measure. However, there is, up to now, no guarantee to distinguish structural modes from spurious ones when deal with complex structure with noisy measurements.

On the other hand, classical frequency domain (FD) techniques, such as PSD peak picking, have been applied for OMA. The PP technique gives reasonable modal estimates if the modes are well separated [10].The main advantages compared to the TD techniques are that it has no bother of computational modes and is much faster and simpler to use. However, PSD peak picking technique is normally inaccurate. The accuracy of modal frequency estimation is limited to the frequency resolution of the PSD spectrum, operational deflection shapes is obtained instead of real mode shapes, damping ration estimation via half-power point is biased or even impossible. Moreover, PP technique is very difficult, if not impossible in dealing with closely spaced modes, which is often encountered for the OMA with real world complex structures.

A challenge was raised if we could develop a FD technique that has all the advantages but doesn't have the disadvantages of the PSD peak picking technique. A new FD operational modal identification technique, named as Frequency Domain Decomposition (FDD) was then developed in 2000 to answer the challenge [11]. The first generation of FDD technique was proposed for estimation of modal frequencies and mode shapes. Enhance FDD is then developed to extend to damping ratio extraction [12]. Recently, a frequency-spatial domain decomposition (FSDD) has been proposed to further improve the FDD performance.

Theoretical background for FSDD techniques is described in this paper, followed by applications of the FSDD for OMA of civil engineering structures. Three typical applications, i.e. a. large-span stadium roof for verifying finite-element model, a highway bridge for damage detection and a long-span cable-stayed bridge for structural health monitoring, are presented.

2. Frequency Domain Decomposition Technique

2.1 OMA in Frequency Domain, PSD-based Procedures (I)

Frequency domain decomposition technique is based on the formula of input and output PSD relationship for stochastic process.

$$G_{yy}(j\omega) = H(j\omega)^* G_{xx}(j\omega) H(j\omega)^T$$

Where $G_{xx}(j\omega)$, $G_{yy}(j\omega)$ are input and output PSD matrix, respectively, $H(j\omega)$ is the FRF matrix, which can be expressed as partial fractions form via poles λ_r and residues R_r,

$$H(j\omega) = \sum_{r=1}^{N} \left(\frac{R_r}{j\omega - \lambda_r} + \frac{R_r^*}{j\omega - \lambda_r^*}\right)$$

Where $\mathrm{R}_r = \phi_r \gamma_r^T$, ϕ_r, γ_r are mode shape and modal participation vector, respectively. When all output measurements are taken as reference, then $H(j\omega)$ are square matrix and $\gamma_r = \phi_r$.

Assuming input is white noise process, i.e. $G_{xx}(j\omega)$ equals to constant, the modal decomposition of the output PSD matrix $G_{yy}(j\omega)$ can be derived as

$$G_{yy}(j\omega) = \sum_{r=1}^{N} \left(\frac{A_r}{j\omega - \lambda_r} + \frac{A_r^H}{-j\omega - \lambda_r^*} + \frac{A_r^*}{j\omega - \lambda_r^*} + \frac{A_r^T}{-j\omega - \lambda_r} \right)$$

Where r^{th} pole $\lambda_r = -\sigma_r + j\omega_{dr}$, corresponding r^{th} residue $A_r \approx d_r \phi_r^* \phi_r^T$, $d_r = \gamma_r^H G_{xx} \gamma_r$, is a real scalar in the case of white noise excitation. In the vicinity of a modal frequency the PSD can be approximated as

$$\underset{\omega \to \omega_r}{G_{yy}^T}(j\omega) \approx \phi_r \Re e\left(\frac{2d_r}{j\omega - \lambda_r} \right) \phi_r^H = \alpha_r \phi_r \phi_r^H$$

Classical Frequency Domain (FD) approach is Peak Picking technique (PP). PP is based on the fact that modal frequencies directly from the Power Spectral Density (PSD) plot at the peak, and mode shapes can be obtained as a column of the PSD matrix at the corresponding damped natural frequency.

The key of the FDD technique is to conduct singular value decomposition (SVD) of output PSD, estimated at discrete frequencies $\omega = \omega_i$,

$$\hat{G}_{yy}(j\omega_i) = U_i S_i V_i^H$$

Where $\boldsymbol{S_i}$ is a diagonal matrix, consisting of scalar singular values $\boldsymbol{s_{ij}}$ $U_i = [u_{i1}, u_{i2}, \ldots, u_{im}]$, $V_i = [v_{i1}, v_{i2}, \ldots, v_{im}]$ are corresponding right and left unitary matrices, consisting of the singular vectors $\boldsymbol{u_{ij}}$,$\boldsymbol{v_{ij}}$, respectively. When all output measurements are taken as references, then U_i=V_i. It is observed that singular values are the function of the frequency. When the frequency approaches to a modal frequency ω_r, the PSD matrix approximates to a rank one matrix as

$$\hat{G}_{\substack{yy \\ \omega_i \to \omega_r}}(j\omega_i) = s_i u_{i1}\, u_{i1}{}^H$$

The first singular value (SV) reaches maximum. The corresponding singular vector u_{r1} is an estimate of the r^{th} mode shape $\hat{\phi}_r = u_{r1}$ with unitary normalization. In the repeated mode case, the rank of PSD matrix will be equal to the number of multiplicity of the modes. Therefore, the SV function can suitably be adopted as modal indication function (MIF). Modal frequencies can be located by the peaks of the SV plots. From the corresponding singular vectors, mode shapes can be obtained. Since SVD has the ability of separating signal space from noise space, the modes can be indicated from SV plots with noisy measurements, and closely spaced modes or even repeated modes can easily be detected.

The **first generation** of FDD can only estimate modal frequencies and mode shapes. The **second generation** of FDD, which is called as Enhanced FDD or EFDD has been followed for estimation of not only modal frequencies and mode shapes, but damping ratios [12].To do so, the singular value data near the peak with corresponding singular vector having enough high MAC value are transferred back to time domain via inverse FFT, which is approximation of correlation function of the SDOF system. From this free decay function of the S-DOF system, the modal frequency and the damping ratio are then calculated by the logarithmic decrement (Logdec) technique.

Since only truncated data, i.e. the data near the peak of the SV plot are used for the inverse FTT to calculate approximate correlation function of the corresponding S-DOF system. It may cause bias error in damping estimation. Moreover, when dealing with closely spaced modes, beat phenomena would be encountered, which can leads inaccurate estimation of damping ratio by Logdec technique.

The **third generation** of FDD, i.e. Frequency-Spatial Domain Decomposition (FSDD), has been developed recently to eliminate these shortcomings. FSDD makes use of the property of unitary singular matrix to derive an enhanced output PSD via pre and post-multiplying a singular vector corresponding to the r^{th} damped natural frequency,

$$\hat{G}^{Enh}_{\substack{yy \\ \omega \to \omega_r}}(j\omega) = u_{r1}^H \hat{G}_{yy}(j\omega) u_{r1} \approx \Re e\left(\frac{2d_r}{j\omega - \lambda_r}\right)$$

It is seen that the output PSD is enhanced in the vicinity of the r^{th} modal frequency and behaves like an S-DOF system, and beyond this region it is attenuated. In other words, the singular vector corresponding to a modal frequency acts as a modal filter. In most cases the enhanced PSD can be approximated as S-DOF system, and therefore an S-DOF curve fitter making use of the spectral lines in the vicinity of a mode can be adopted to estimate relevant modal frequency and damping ratio.

3. Application of FSDD to OMA of civil Engineering Structures

3.1 Application to a Large-span Roof Structure

An ambient modal testing was conducted with respect to the roof of the Tokyo Horse Raising Stadium (HRS) right after finishing remodeling of the 1/3 of the stadium. The roof measures 108×49 meters (Figure 1). 66 accelerometers were rather uniformly placed in the square-shaped roof with 4 setups to measure vertical vibration. Three of them were used as references. Although there are many modes in the frequency range of interest, high quality MIF was obtained to show clearly all the structural modes, see Figure 2. 16 modes are then identified within 5.6 Hz. Figure 3 shows relevant mode shapes.

Figure (1a) The Tokyo Horse Raising Stadium (1/3 part)

Figure (1b) The roof of the Stadium

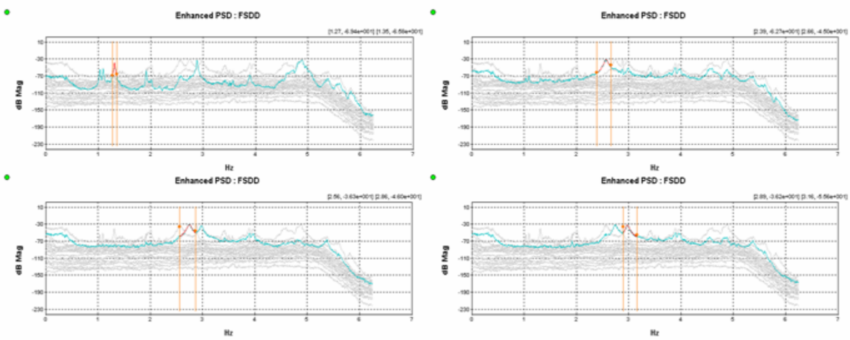

Figure 2 Modal Indication Function of the HRS roof

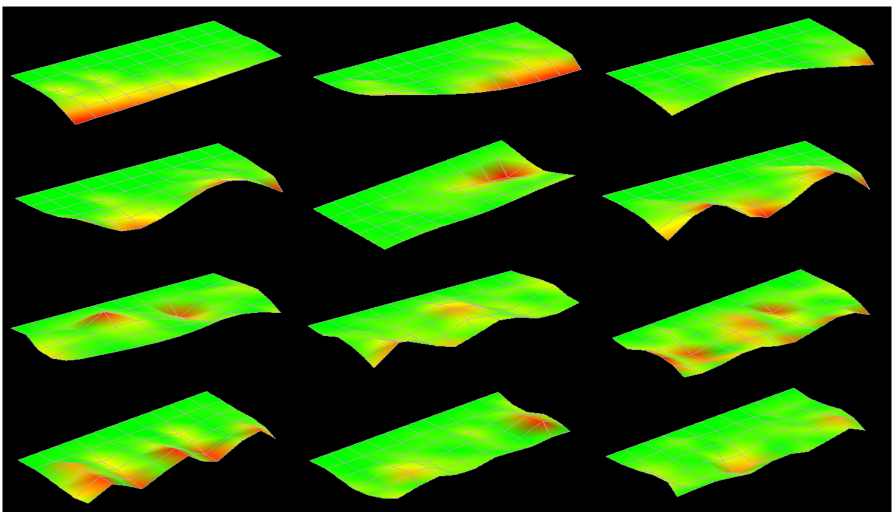

Figure 3 FSDD-Identified mode shapes of the HRS roof

3.2 Application to a Highway Bridge

Ambient response measurements of a well known Z24 bridge are applied as a case study for FSDD operational modal identification. Z24 ridge is an old Swiss bridge over passing the national highway between Bern and Zurich. It is a traditional pre-stressed concrete box girder bridge with main span of 30 m and two side spans of 14 m, and supported by 4 piers clamped into the girders (Figure 4). (1)).

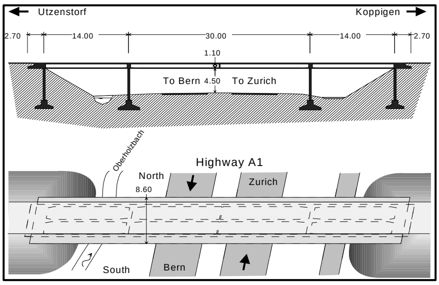

Figure 4   Z-24 Bridge

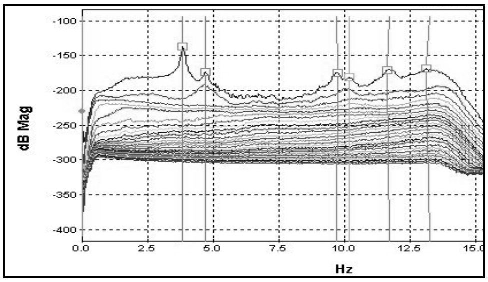

Figure 5   MIF of the Z-24 Bridge

OMA was conducted utilizing response measurements subject to traffic excitation. The data was obtained from 9 accelerometer setups, 8 sets with 33 channels and one with 27 channels. Three reference sensors were adopted including one unidirectional and one 3D sensor.

From singular value plot, which acts as modal indication function (MIF) shown in Figure 5, it can be clearly seen that 6 modes in the frequency range of interest are well separated from noisy measurements. Enhanced PSD is then computed making use of 6 singular vectors corresponding to the 6 peak spectral lines as modal filters (Figure 6). Figure 7 gives identified 6 mode shapes.

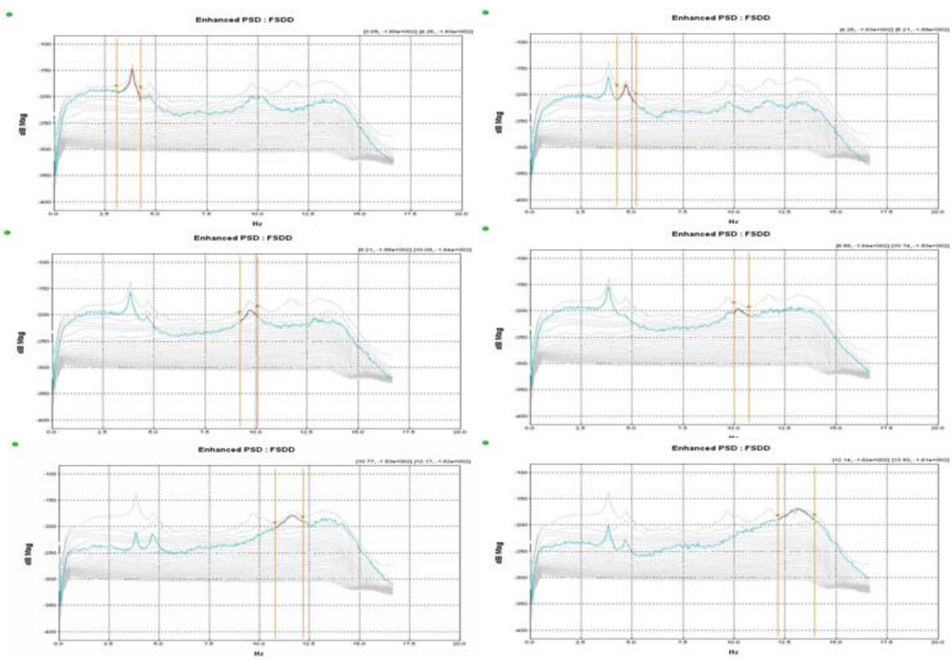

Figure 6   Enhanced PSD and curve fitting of the Z-24 bridge

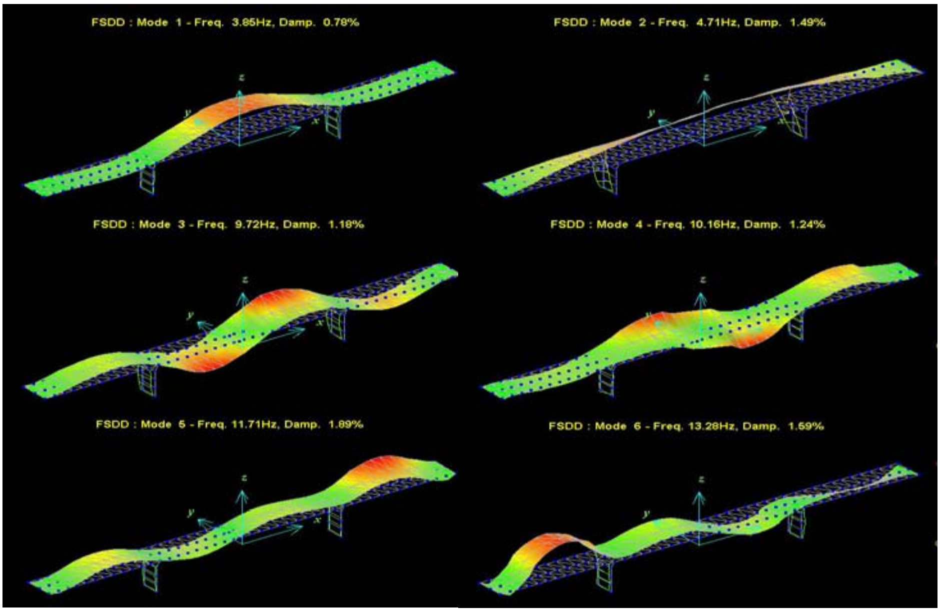

Figure 7 FSDD-Identified mode shapes of the Z-24 bridge

3.3. Application to A long-span Cable-stay Bridge

The Ting Kau bridge is a cable-stayed bridge with two main spans of 475 and 485 m, respectively, located in Hong Kong (Figure 8). More than 200 sensors, including 30 accelerometers, are permanently installed inside the bridge for structural health monitoring. .24 accelerometers are placed in 8 sections of the bridge, two vertical and one transversal direction at each section. Four accelerometers set on the center tower with three in transversal and one in vertical direction. Two accelerometers with one each in transversal are placed at the two site towers. One monitoring data for each channel sampled at rate of 25.6 Hz was utilized for OMA via FSDD. Decimation of 6 leads to maximum frequency of 1.67 Hz.

It is hardly to tell how many modes exist in the frequency range from PSD plots, see Figure 9. However, MIF based on SVD of response PSD matrix can still indicate clearly the structural modes. Favorable enhanced PSD can also be obtained for almost all the modes. Due to the limitation of the size of the paper, only a few enhanced PSD plots are shown in Figure 10.All together 54 modes are successfully identified from these monitoring data of the bridge under operational condition.

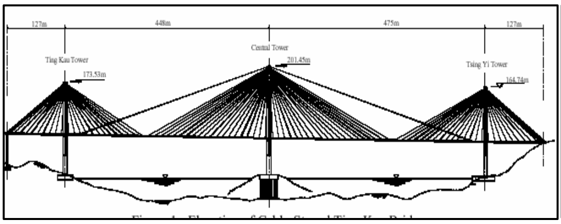

Figure 8 Schematic plot of the Ting Kau bridge

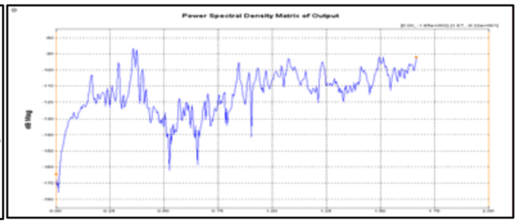

Figure 9 PSD obtained from monitored response

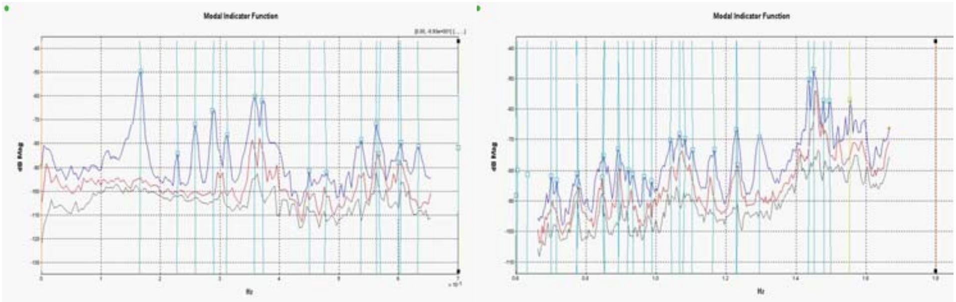

Figure 10 MIF of the cable-stayed Ting Kau bridge

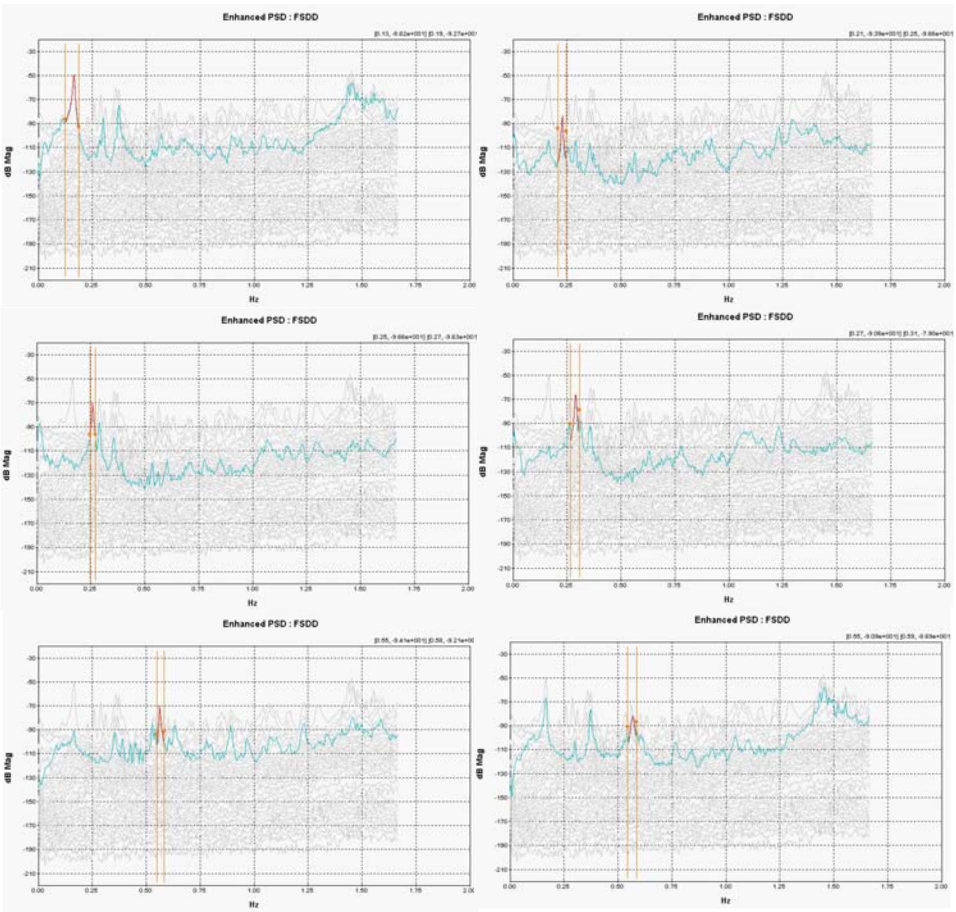

Figure 11 Enhanced PSD and curve fitting of the Ting Kau bridge

4. Conclusion

The development and theoretical background of the new operational modal identification technique FSDD is described in this paper. Three applications to the civil engineering structures for typical purposes are presented, i.e. a. large-span stadium roof for verifying finite-element model, a highway bridge for damage detection and a long-span cable-stayed bridge for structural health monitoring. Favorable results have been obtained via new developed operational modal identification technique frequency-spatial domain decomposition (FSDD). It is shown that structural modes can clearly seen by the modal indication functions obtained from singular value plot computed from output PSD measurements. FSDD is actually narrow band FD identification approach with S-DOF curve fitting with the capability of dealing with very closely spaced or even repeated modes. An enhanced FSDD is underway, which makes use of two singular vector to compute enhanced PSD, and therefore, the accuracy of curve fitting with two closely spaced modes can be further improved via two-DOF curve fitter.

5. References

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