Load Estimation from Natural input Modal Analysis

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NOMENCLATURE

Stiffness matrix	$[k]$	Mass matrix	$[m]$
Damping matrix	$[c]$	Force vector	$\{f(t)\}$
FRF matrix	$[H(\omega)]$	Spectral matrix	$[S(\omega)]$
Natural frequency	ω_r	Scaled mode shape	$\{\phi\}_r$
Un-scaled mode shape	$\{\psi\}_r$	Scaling factor	α_r
Damping factor	ζ_r		

ABSTRACT

One application of Natural Input Modal Analysis consists in estimating the unknown load acting on structures such as wind loads, wave loads, traffic loads, etc. In this paper, a procedure to determine loading from a truncated modal model, as well as the results of an experimental testing programme, are presented. The method involves the inversion of the FRF matrix partly solving the numerical problems that appear because of the truncation of the modal space. However, the error in the load estimation depends on the degree of truncation of the modal space and on the accuracy of the modal parameter estimation. In the experimental program a small structure subjected to vibration was used to estimate the loading from the measurements and the experimental modal space. The modal parameters were estimated by Natural Input Modal Analysis and the scaling factors of the mode shapes obtained by the mass change method [2] [3] [4]. Finally, the calculated loading is compared with the actual loading in order to determine the magnitude of the errors.

1. INTRODUCTION

When Natural Input Modal testing is performed, the testing is normally done by just measuring the responses under the natural conditions. This means that the forces are normally not recorded or controlled. This represents an important advantage compared with traditional modal analysis, mainly for big structures. Wind loads, wave loads, traffic loads, etc. can be considered natural input.

On the other hand, engineers are interested in obtaining information about those loads which are difficult to measure directly (wind loads, wave loads, traffic loads, etc.) . They can also learn about the correlation length in the loading and thus learn how to perform better natural input modal analysis [1].

In this paper, a procedure to determine loading from a truncated modal model, as well as the results of an experimental testing programme, are presented. This procedure involves the inversion of the FRF matrix; also other numerical problems appear because of the truncation of the modal space.

An experimental program is carried out to estimate the loading from the measurements and the experimental modal space. A steel cantilever beam are used. The modal parameters are obtained by natural input modal analysis and the scaling factors of the mode shapes by the mass change method [2] [3] [4]. Finally, the estimated loading is compared with the actual loading in order to determine the magnitude and the sources of the error.

2. THE METHOD

As well know, the equation of motion of a structure subjected to a force $\{f(t)\}$ is given by:

$$[m]\cdot\{\ddot{u}\}+[c]\cdot\{\dot{u}\}+[k]\cdot\{u\}=\{f(t)\} \quad (1)$$

Transforming equation (1) in frequency domain by Fourier, yields:

$$\left(-\omega^2\cdot[m]+j\omega\cdot[c]+[k]\right)\cdot\{U(\omega)\}=\{F(\omega)\} \quad (2)$$

Defining the frequency response function matrix (FRF) or transfer function matrix as:

$$[H(\omega)]=\left(-\omega^2\cdot[m]+j\omega\cdot[c]+[k]\right)^{-1} \quad (3)$$

and substituting the equation (3) in equation (2), it results:

$$\{U(\omega)\}=[H(\omega)]\cdot\{F(\omega)\} \quad (4)$$

the loading in frequency domain can be calculated by:

$$\{F(\omega)\}=[H(\omega)]^{-1}\cdot\{U(\omega)\} \quad (5)$$

The force in time domain can be obtained applying the inverse Fourier transform of $\{F(\omega)\}$. The process is schematically shown in Figure 1.

To solve equation (5), the FRF matrix and the responses have to be known. If modal analysis is performed, the responses are measured and the modal parameters can be estimated. Subsequently, the FRF can be constructed from the modal parameters.

The spectral density function load matrix can then be obtained from the spectral density function response matrix by means of the expression:

$$[S_{FF}(\omega)]=[H(\omega)]^{-1}\cdot[S_{UU}(\omega)]\cdot[H(\omega)]^{-H} \quad (6)$$

where the superscript ^{H} denotes complex conjugate transpose.

3. THE FRF MATRIX

When the FRF matrix is obtained from the modal parameters, the following information is needed for each mode:

- The natural frequencies ω_r ,
- The damping factors ζ_r ,
- The mass normalized (scaled) mode shape $\{\phi_r\}$.

However, when natural input modal analysis is performed, the forces are unknown so that only the following information can be obtained for each mode:

- The natural frequencies ω_r ,
- The damping factors ζ_r ,
- The un-scaled mode shape $\{\psi_r\}$.

The scaled and un-scale mode shapes are related by

$$\{\phi_r\} = \alpha_r \cdot \{\psi_r\} \tag{7}$$

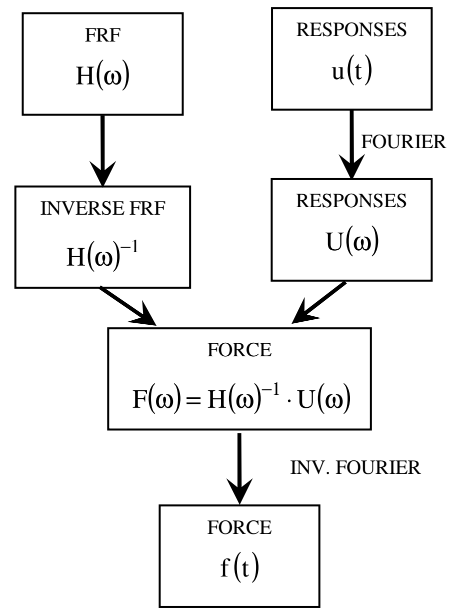

Fig. 1. Process to estimate the loading

where α_r is the scaling factor of the r-th mode.

Therefore, when natural input modal analysis is used, an extra method is needed to calculate the scaling factors. Recently, different methods has been proposed to estimate the scaling factors involving repeated testing in which mass changes are introduced in the points where the mode shapes are known[2] [3] [4].

In case of complex modes, the expression of the FRF matrix when the modal space is used, is given by[5] :

$$[H(\omega)] = \sum_{r=1}^{N} \left(\frac{Q_r \{\psi_r\} \cdot \{\psi_r\}^T}{j\omega - \lambda_r} + \frac{Q_r^* \{\psi_r\}^* \cdot \{\psi_r\}^{*T}}{j\omega - \lambda_r^*} \right) \tag{8}$$

where:

- $\{\psi_r\}$ is the r-th un-scaled mode shape,
- $\lambda_r = -\zeta_r \omega_r + j\omega_r \sqrt{1-\zeta_r^2}$ is the pole of the r-th mode
- Q_r is a factor which takes into account the scale of the mode, and
- the superscript * denotes complex conjugate

The factor Q_r can be related to α_r through [5]:

$$Q_r = \frac{\alpha_r^2}{2j\omega_r} \tag{9}$$

4. INVERSION OF THE FRF MATRIX

Equations (5) and (6) involve the inversion of the FRF matrix frequency by frequency, but this inversion can only be performed using standard methods when the FRF matrix is full rank, i.e., when the number of modes is equal to the number of observation points. Otherwise, when a truncated modal space is used, the FRF matrix is singular and normal inverse does not apply anymore. However, the inversion of the FRF matrix can still be done using singular value decomposition (SVD).

The singular value decomposition of a complex matrix $[H]$ is:

$$[H] = [U] \cdot [\Sigma] \cdot [V]^H \qquad (10)$$

where:

- $[U]$ and $[V]$ are unitary matrices (orthogonal in case of real matrices), and
- $[\Sigma]$ is a diagonal matrix containing the singular values. The number of non-zero singular values are equal to the number of modes active at the considered frequency.

The superscript ^{H} denotes complex conjugate transpose.

Using equation (10) the inverse of the matrix $[H]$ can be obtained as:

$$[H]^{-1} = [V]^{-H} \cdot [\Sigma]^{-1} \cdot [U]^{-1} \qquad (11)$$

Taking in account the properties of unitary matrices, i.e.:

$$[U]^H = [U]^{-1} \quad \text{and} \quad [V]^H = [V]^{-1} \qquad (12)$$

the equation (11) becomes:

$$[H]^{-1} = [V] \cdot [\Sigma]^{-1} \cdot [U]^H \qquad (13)$$

where only the non-zero singular values must be used in the calculation.

Equation (13) provides the exact solution when all modes are considered, but due to the truncation effect, the calculated FRF matrix and its inverse will only represent an approximation. As soon as more modes are considered, better accuracy will be achieved.

5. LEAKAGE REDUCTION.

Equations (5) and (6) involve the Fourier transform of the responses so that the analysis of a finite time record can cause leakage. A method to minimize the leakage effect is to apply a window.

The application of a window spreads some of the energy of the original signals to the adjoining spectral components while it suppresses the energy leaked to other spectral components which are far from the correct frequency.

When a window is used to obtain the spectrum, some information gets lost and the original signal in time domain can not be recovered by inverse Fourier transform of the corresponding spectrum. For this reason, if the force in time domain is the objective, windows can not be applied and leakage errors will be present in the estimated force, but If the objective is the spectral density function force matrix, a window should be used when the Fourier transform of the responses are calculated.

However, leakage can still be reduced using the method described in the next section for real time estimation.

6. REAL TIME ESTIMATION.

If the modal parameters of the structure are known and the responses are measured in real time, then the force can also be estimated in real time. Taking N points of the recorded response, the force corresponding to this segment can be estimated using equation (5).

It we want to calculate the force corresponding to N points, a way to reduce the leakage is to calculate the force for a larger segment, i.e., a segment of βN points, where $\beta > 1$ and then select the central N points. Thus, the $\frac{(\beta - 1)N}{2}$ points of the estimated load on both the right and left sides are discarded. In Figure 2, the load calculation for $\beta = 2$ is shown .

With this method the leakage will be reduced. The only inconvenient is that more numerical operations have to be performed to obtain the load.

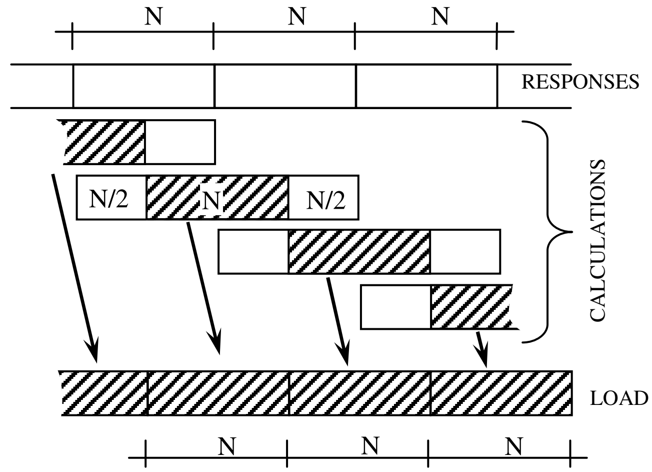

Fig 2. Real time load estimation with $\beta = 2$

7. ERRORS.

There are four main sources of error when the load is estimated:

- Errors due to noise in the responses
- Errors in the modal identification.
- Errors due to the truncation of modes
- Errors due to leakage.

When performing experimental measurements, some noise is always present in the signals. This type of error can be reduced using better sensors but can not be removed. The noise effect can also be reduced by filtering. A singular value decomposition of the responses can help to decide which frequencies should be filtered.

The errors in the modal parameters estimation depend on several factors such us the level of noise in the responses, the method and the software used in the estimation, the type of excitation, etc.

As mentioned before, when the modal space is truncated only an approximation of the FRF matrix can be obtained so that these errors will be amplified when the inverse operation are performed.

Finally, as also pointed out previously, the errors due to leakage can be reduced if the objective is the calculation of the spectral density force matrix, but they will be present when the force in time domain is the objective.

8. EXPERIMENTAL TESTS

A steel cantilever beam was used to perform the tests. The beam was 1.85 m length, with a 80x50x4 tube rectangular section and the responses were measured in 8 degree of freedoms regularly distributed along the beam (Figure 3). Two types of excitation were used: Stationary broad banded and impact.

8.1 Stationary broad banded tests

In order to determine the modal parameters, a stationary broad banded excitation was applied to the structure. The loading was not measured so that a natural input modal analysis software was used to estimate the natural frequencies, the mode shapes and the damping. Only the first 5 modes were considered in the analysis.

The scaling factors were determined by means of the mass change method [2] [3] [4]. A proportional mass change was applied to the structure attaching masses of 180 grams in each degree of freedom (except at the free border of the cantilever beam at which a 90 gram mass was attached). Finally, a new stationary broad banded excitation was applied to the modified structure.

The modal parameters are shown in table 1. As can be seen, the damping is very low. The scaling factors shown in the table correspond to mode shapes normalized to unity.

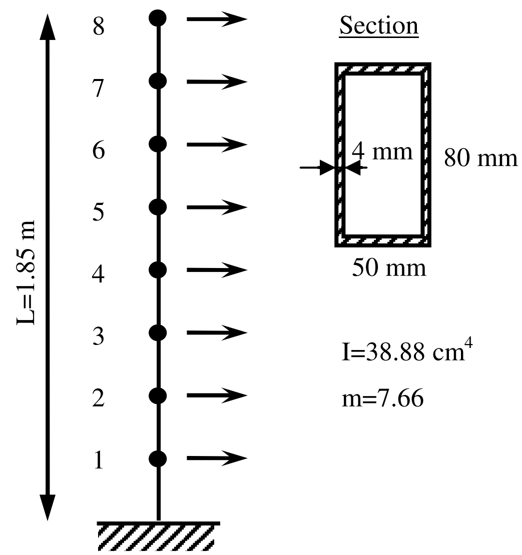

Fig 3. Cantilever beam.

Table 1. Modal parameters of the cantilever beam.

Mode	1	2	3	4	5
Natural frequencies (Hz)	15.65	97.46	269.71	517.41	830.43
Damping (%)	0.24	0.13	0.18	0.10	0.12
Scaling factors	0.450	0.425	0.391	0.354	0.354

8.2 Impact tests

In order to check the accuracy of the proposed method, several impacts were applied in each degree of freedom of the unmodified structure. Only one hit was applied every time. An impact hammer with a rubber tip was used to apply the hits. The responses, together with the FRF matrix, were used to estimate the load with the method proposed in this paper.

The real force in time domain applied on the 8th degree of freedom is shown in Figure 4a whereas the estimated force is shown in Figure 4b. A zoom of the impact is shown in Figure 5. As can be observed, the impacts can be detected and the error is reasonable low. Due to the noise present in the responses and the errors in the estimated FRF matrix (amplified in the inversion process) a low level force is estimated in all channels (Figure 4b). Furthermore, when the impact force is applied on a degree of freedom, a small peak is estimated in the adjacent degree of freedoms. In Figure 4b it can be seen that a small peak is estimated in the 8th degree of freedom when the impact is applied on the 7th degree of freedom.

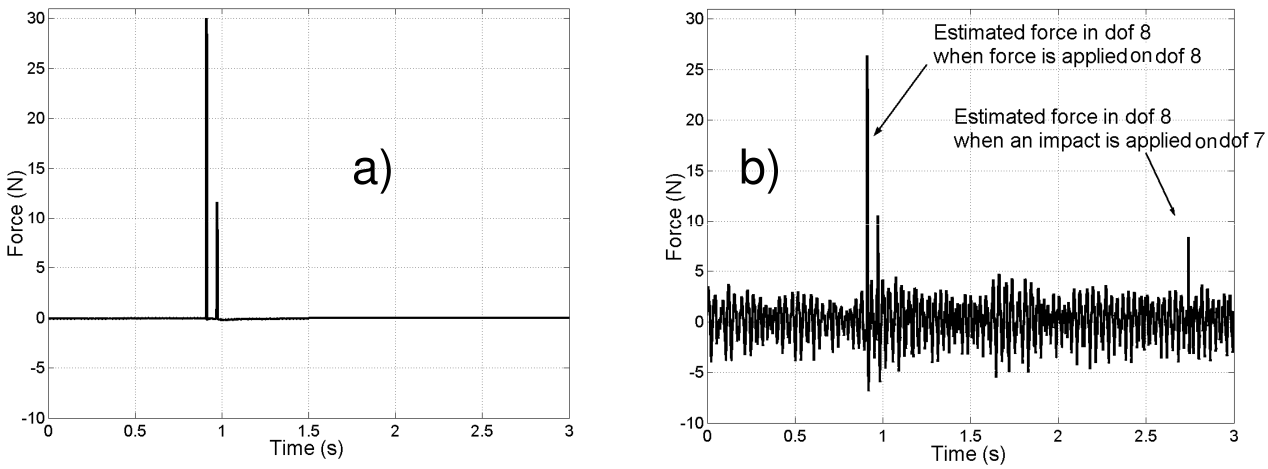

Fig 4. a) Real force applied in the 8th degree of freedom. b) Estimated force in the same degree of freedom.

Due to the fact that the damping is very low, a large number of points (30000 points) had to be used to reduce the leakage effect. Furthermore, a low pass filter was applied to reduce the noise effect at low frequencies present in the responses. For this reason, the error in the magnitude of the estimated force in time domain is significant.

The error in the magnitude of the estimated force is low at the free border of the beam (approximately 10% of the exact magnitude) whereas the error increases as the degree of freedom is nearer the support where the maximum error is approximately 40% of the exact magnitude.

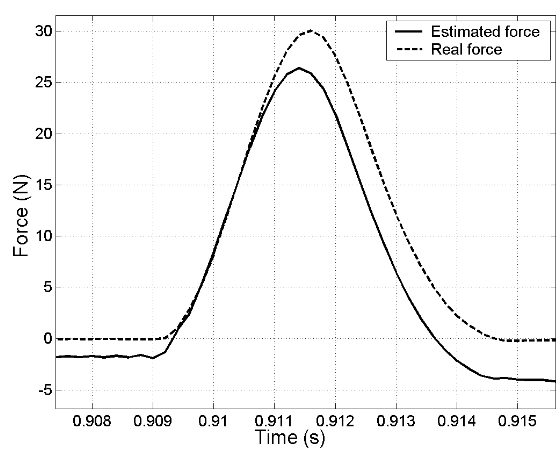

Fig 5. Zoom of the real and estimated peak in the 8th degree of freedom

The force autospectral density of both the estimated and the recorded force, corresponding to the 8th degree of freedom is shown in Figure 6. Windows to reduce the leakage effect were not applied. As can be seen, the estimation is reasonable good excepting the peaks in the resonances which appear due to the errors in the modal parameters estimation.

The force estimation can be improved removing or reducing the peaks in the spectrum by means of a smoothing technique, but it was not used in this paper.

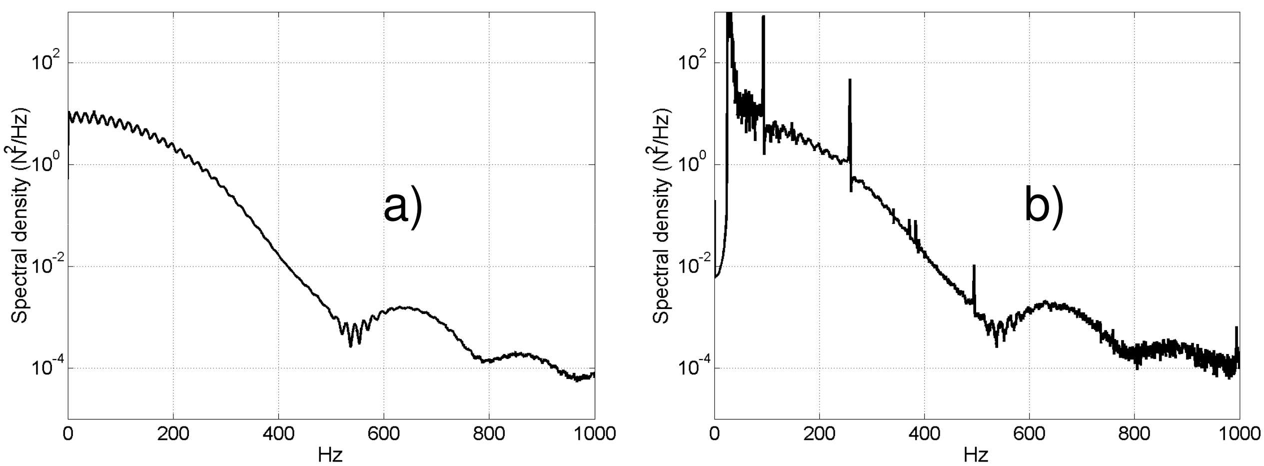

Fig 6. a) Real force spectral density in the 8th degree of freedom. b) Estimated spectral density in the 8th degree of freedom.

9. CONCLUSSIONS

- A procedure is proposed to estimate the loading exciting a structure, from the experimental responses and the modal parameters estimated by Natural Input Modal Analysis.
- A steel cantilever beam is used to check the method proposed in this paper. The modal parameters are estimated by natural input modal analysis which are used together with the experimental responses to estimate the impact forces applied to the structure with an impact hammer.
- The estimated force in frequency domain is quite good. Although the errors in the estimation of modal parameters, some peaks appear in the spectra at the resonances.
- A low pass filter has to be used to remove the noise at low frequencies. For this reason, the force in time domain can not be estimated accurately.

10. ACKNOWLEDGEMENTS

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