Eliminating the Influence of Harmonic Components in Operational Modal Analysis

Niels-Jørgen Jacobsen
Brüel & Kjær Sound & Vibration Measurement A/S
Skodsborgvej 307, DK-2850 Nærum, Denmark

Palle Andersen
Structural Vibration Solutions A/S
NOVI Science Park, Niels Jerners Vej 10, DK-9220 Aalborg East, Denmark

Rune Brincker
Department of Civil Engineering, University of Aalborg
Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark

ABSTRACT

Operational modal analysis is used for determining the modal parameters of structures for which the input forces cannot be measured. However, the algorithms used assume that the input forces are stochastic in nature. While this is often the case for civil engineering structures, mechanical structures, in contrast, are subject inherently to deterministic forces due to the rotating parts in the machinery. These forces are seen as harmonic components in the responses, and their influence should be eliminated before extracting the modes in their vicinity.
This paper describes a new method based on the well-known Enhanced Frequency Domain Decomposition (EFDD) technique for eliminating these harmonic components in the modal parameter extraction process. For assessing the quality of the method, various experiments were carried out where the results were compared with those obtained with pure stochastic excitation of the same structure. Good agreement was found and the method is shown to be an easy and robust tool for enhancing the EFDD technique for mechanical structures.

1 INTRODUCTION TO OPERATIONAL MODAL ANALYSIS

Operational Modal Analysis (OMA) is based on measuring the output of a structure only and using the ambient and natural operating forces as unmeasured input. It is used instead of classical mobility-based modal analysis for accurate modal identification under actual operating conditions, and in situations where it is difficult or impossible to artificially excite the structure.

The algorithms used in OMA assume that the input forces are stochastic in nature. This is often the case for civil engineering structures like buildings, towers, bridges and offshore structures, which are mainly loaded by ambient forces like wind, waves, traffic or seismic micro-tremors. The loading forces of many mechanical structures are, however, often more complex. They are typically a combination of harmonic components (deterministic signals) originating from the rotating and reciprocating parts and broadband excitation originating from either self-generated vibrations from, for example, bearings and combustions or from ambient excitations like air turbulence and road vibrations. However, civil engineering structures can also have broadband responses superimposed by harmonic components from, for example, ventilation systems, turbines and generators.

As the input forces to the structure are not measured in OMA, special attention must be paid to identify and separate harmonic components from true structural modes and eliminate the influence of the harmonic components in the modal parameter extraction process.

This paper starts by describing the consequences of having harmonic components present in the responses for different modal parameter identification techniques. Then an overview of various methods for separating harmonic components from structural modes is given, before explaining in more depth a very easy-to-use and robust method based on kurtosis. The well-known and popular Enhanced Frequency Domain Decomposition (EFDD) modal identification technique is briefly explained, leading to a description of the new EFDD-based method for eliminating the influence of harmonic components in OMA.

Finally the quality of the method is assessed from various experiments using a plate structure. The structure is excited by a combination of a single sinusoidal signal and a broadband stochastic signal and compared to the results obtained with pure stochastic excitation of the same structure.

2 CONSEQUENCES OF HARMONIC COMPONENTS

The consequences of having harmonic components present in the responses depend on both the nature of the harmonic components (number, frequency, and level) and the modal parameter extraction method used. Table 1 below indicates these consequences for the Frequency Domain Decomposition (FDD), Enhanced FDD (EFDD) and Stochastic Subspace Identification (SSI) techniques. See [1], [2] for further description of the FDD, EFDD and SSI techniques. The EFDD method is briefly explained in Chapter 4.2 as well.

Technique	Consequences
All techniques	• Harmonic components are potentially mistaken for being structural modes • Harmonic components might potentially bias the estimation of the structural modes (natural frequency, modal damping, mode shape) • A high dynamic range might be required to extract "weak" modes
FDD	• The picked FFT line might be biased by the harmonic component(s) • Harmonic components must be away from the structural modes (only the picked FFT line is used in the FDD technique)
EFDD	• The identified SDOF function used for modal parameter estimation might be biased by the harmonic component(s) • Harmonic components must be outside the SDOF function thereby potentially narrowing the SDOF function and resulting in poorer identification (leakage)
SSI (PC, UPC, CVA)	• The SSI methods estimate both harmonic components and structural modes. The modes are estimated correctly even for harmonic components close to - or with equal frequency as - the modes • Information in the time signal is used both to extract the harmonic components and the modes, therefore the recording time should generally be longer

Table 1. Consequences of harmonic components for various identification techniques.

For the EFDD technique it is important that harmonic components inside the desired SDOF function are identified and their influence eliminated before processing with the modal parameter extraction process.

It should be noticed that harmonic components cannot, in general, be removed by simple filtering as this would in most practical cases significantly change the poles of the structural modes and thereby their natural frequency and modal damping.

3 IDENTIFICATION OF HARMONIC COMPONENTS

This chapter gives a brief overview of some useful methods for identifying harmonic components and structural modes in OMA response data followed by a more detailed description of the kurtosis method.

3.1 OVERVIEW OF METHODS

In [3] various methods for identifying harmonic components and structural modes were investigated. A brief overview is presented in Table 2 below.

Technique	Description
Short Time Fourier Transform (STFT)	When responses are shown in a contour plot, structural modes are shown as thick vertical lines. Harmonic components are shown as thin vertical lines for stable conditions (fixed frequencies) and as, for example, “saw tooth” patterns for run-up/down conditions (varying frequencies).
Singular Value Decomposition (SVD)	For a shaped broadband white noise signal exciting the structure, the rank of the matrix containing the singular values will be 1 at frequencies, where only one mode is dominating and higher, if closely-coupled modes or repeated roots are present. In the case of harmonic components, a high rank will be seen at these frequencies, as all modes will be excited. The rank will correspond to the number of responses assuming the dynamic range in the measurement is sufficiently high. Hence, when the SVD curves are plotted, the peaks will indicate whether they are due to a harmonic component or a structural mode.
Visual Mode Shapes Comparison	When a harmonic component is far away from a structural mode, the operating deflection shape (ODS) caused by the harmonic component will be a combination of several excited modes and the loading forces acting on the structure. However, when a harmonic component is close to an isolated structural mode, the ODS of the harmonic component will resemble the mode shape and thus can be mistaken for being a mode shape.
Modal Assurance Criterion (MAC)	As a harmonic component will excite all modes, the MAC value between a true mode shape and an ODS will show high correlation, if the frequency of the ODS is close to the frequency of the mode shape. On the other hand, the MAC value between two closely spaced modes will, in general, show low correlation. The MAC value between two ODS will depend on the modes being excited.
Stabilization Diagram	Using the SSI techniques, a stabilization diagram showing stable, unstable and noise modes is used to select the optimal State Space Dimension. For modes to be classified as stable, they must fulfil certain mode indicator requirements of which one is a valid range of damping ratios. By adjusting this range, both harmonic components and non-physical modes can be filtered out thereby only indicating the true structural modes as stable modes.
Probability Density Functions (PDFs)	The significant difference in the statistical properties of a harmonic component and a narrowband stochastic response of a structural mode can be used as a harmonic indicator. Each potential mode is isolated by band-pass filtering, the PDF is calculated on the result that is subsequently fitted to both the PDF of a pure harmonic component and the PDF of the response of a pure structural mode. The prediction error between the fitted and measured data is calculated in both cases.

Table 2. Overview of methods for identifying harmonic components and structural modes.

3.2 KURTOSIS

In addition to the above-mentioned methods, kurtosis can also be used as a harmonic indicator. The kurtosis γ of a stochastic variable x provides a way of expressing how peaked or how flat the probability density function of x is. The kurtosis is defined as the fourth central moment of the stochastic variable normalised with respect to the standard deviation σ as follows [4]:

$$\gamma(x|\mu,\sigma) = \frac{E\left[(x-\mu)^4\right]}{\sigma^4} \tag{1}$$

where μ is the mean value of x and E is denoting the expectation value.

Often the number 3 is subtracted from equation (1) giving a kurtosis of zero, when x is normally distributed [5]:

$$\gamma^*(x|\mu,\sigma) = \frac{E\left[(x-\mu)^4\right]}{\sigma^4} - 3 \tag{2}$$

Using equation (2), a PDF with a positive kurtosis is said to be leptokurtic. If its kurtosis is negative, it is said to be platykurtic. A PDF with kurtosis equal to zero is called mesokurtic. Leptokurtosis is associated with PDFs that are simultaneously “peaked” and have “fat tails.” Platykurtosis is associated with PDFs that are simultaneously less peaked and have thinner tails.

The PDF of the response of a pure structural mode will be normally distributed, and hence the kurtosis γ^* = 0 (mesokurtic). The PDF y is given by:

$$y = f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{3}$$

A harmonic component can be treated as a stochastic signal if the phase varies randomly within its period. In the case of a pure harmonic component, the PDF will have two distinct peaks approaching infinity at ±a, where a is the amplitude of the harmonic component. The PDF is given by:

$$y = f(x|a) = \left(\pi\cos\left(\arcsin(x/a)\right)\right)^{-1} \tag{4}$$

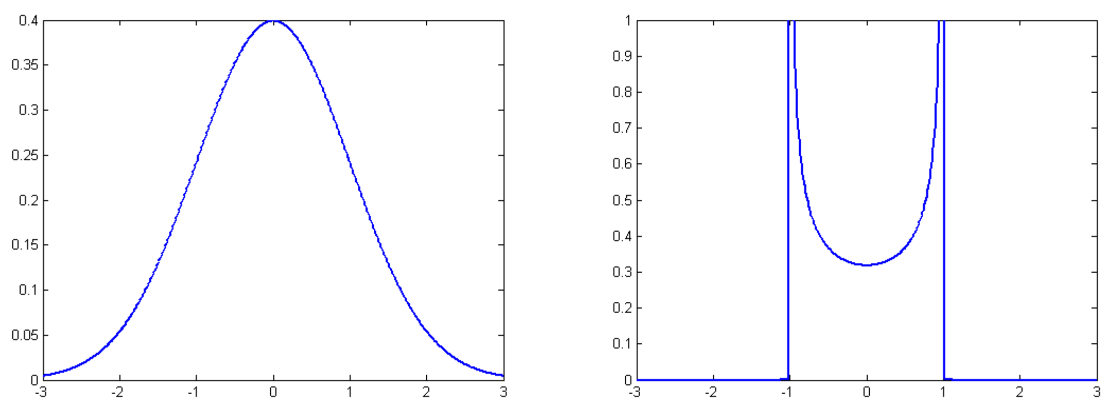

Figure 1. Normalized PDF of the response of a structural mode (left) and a harmonic component (right).

The kurtosis γ^* = -1½ for a sinusoidal component. This difference in kurtosis of various signals is used in the harmonic detection technique described in this paper.

4 ELIMINATING THE INFLUENCE OF HARMONIC COMPONENTS USING THE EFDD TECHNIQUE

Before describing the method, the EFDD technique is briefly explained and the consequences of having harmonic components inside the SDOF function is illustrated.

4.2 BRIEF DESCRIPTION OF THE EXISTING EFDD TECHNIQUE

The Enhanced Frequency Domain Decomposition (EFDD) technique is an extension to the Frequency Domain Decomposition (FDD) technique. FDD is a basic technique that is extremely easy to use. You simply pick the modes by locating the picks in SVD plots calculated from the spectral density spectra of the responses. Animation is performed immediately. As the FDD technique is based on using a single frequency line from the FFT analysis, the accuracy of the estimated natural frequency depends on the FFT resolution and no modal damping is calculated. Compared to FDD, the EFDD gives an improved estimate of both the natural frequencies and the mode shapes and also includes damping.

In EFDD, the SDOF Power Spectral Density function, identified around a resonance peak, is taken back to the time domain using the Inverse Discrete Fourier Transform (IDFT). The natural frequency is obtained by determining the number of zero-crossing as a function of time, and the damping by the logarithmic decrement of the corresponding SDOF normalized auto correlation function. The SDOF function is estimated using the shape determined by the previous FDD peak picking - the latter being used as a reference vector in a correlation analysis based on the Modal Assurance Criterion (MAC). A MAC value is computed between the reference FDD vector and a singular vector for each particular frequency line. If the MAC value of this vector is above a user-specified MAC Rejection Level, the corresponding singular value is included in the description of the SDOF function. The lower this MAC Rejection Level is, the larger the number of singular values included in the identification of the SDOF function will be.

In the left-hand side of Figure 2, the estimated SDOF function for the plate's first bending mode is shown. A pure SDOF function can be extracted. In the right-hand side of Figure 2, a harmonic component close to the resonance frequency is present. As seen, the SDOF function is severely distorted by the presence of the harmonic component and hence the estimation of the modal parameters will be incorrect.

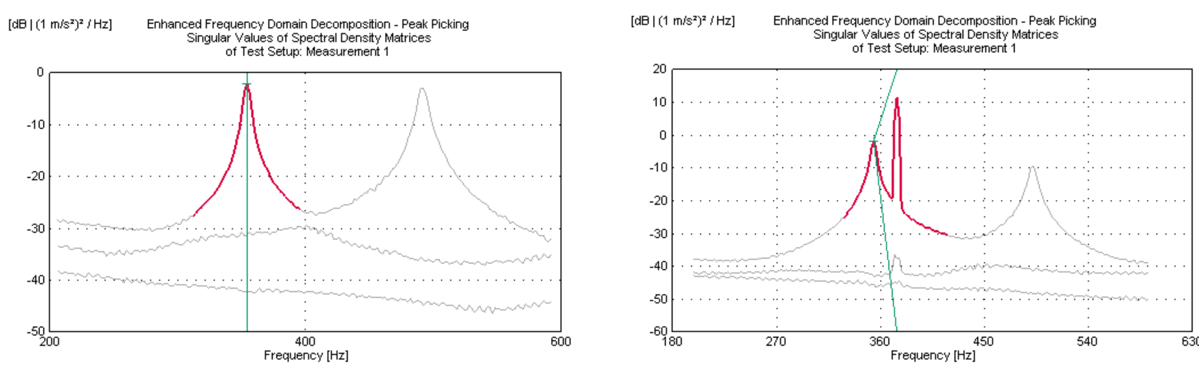

Figure 2. Singular Value SDOF identification without (left) and with (right) harmonic component.
Mode at 354 Hz. Harmonic component at 374 Hz.

Performing an IDFT on the above SDOF functions the normalized correlation functions are calculated. In the upper part of Figure 3, a typical response is seen of a resonating system that decays exponentially. In the lower part of Figure 3, the effect of the harmonic component is clearly visible. The harmonic component can be thought of as a forced vibration with very low damping. The decay is significantly longer and beating phenomena are observed.

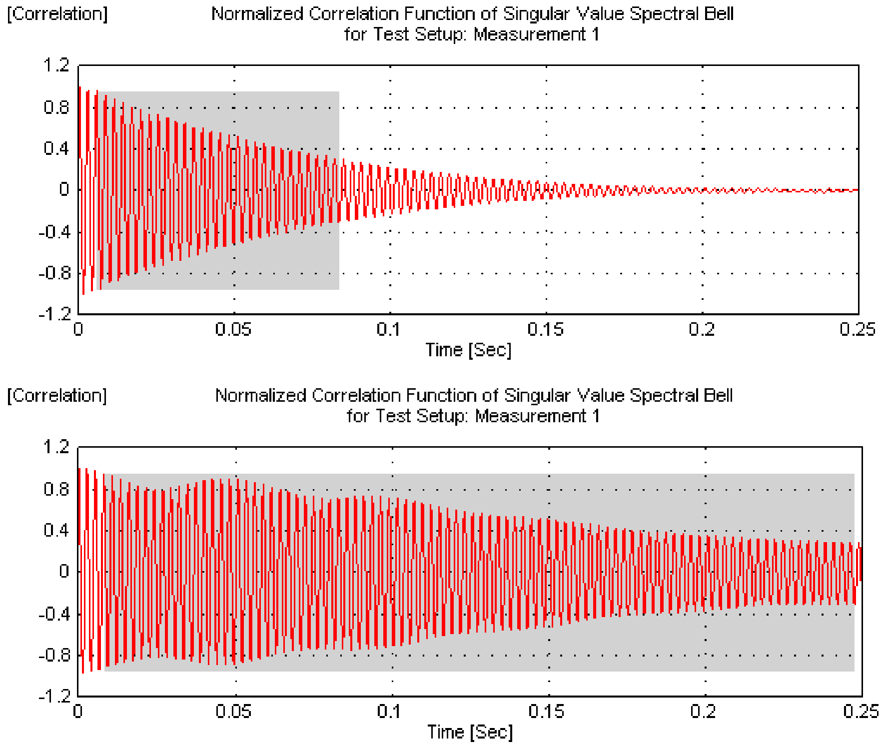

Figure 3. Normalized correlation function without (upper) and with (lower) harmonic component. Mode at 354 Hz. Harmonic component at 374 Hz. Grey area indicates the part of the function used.

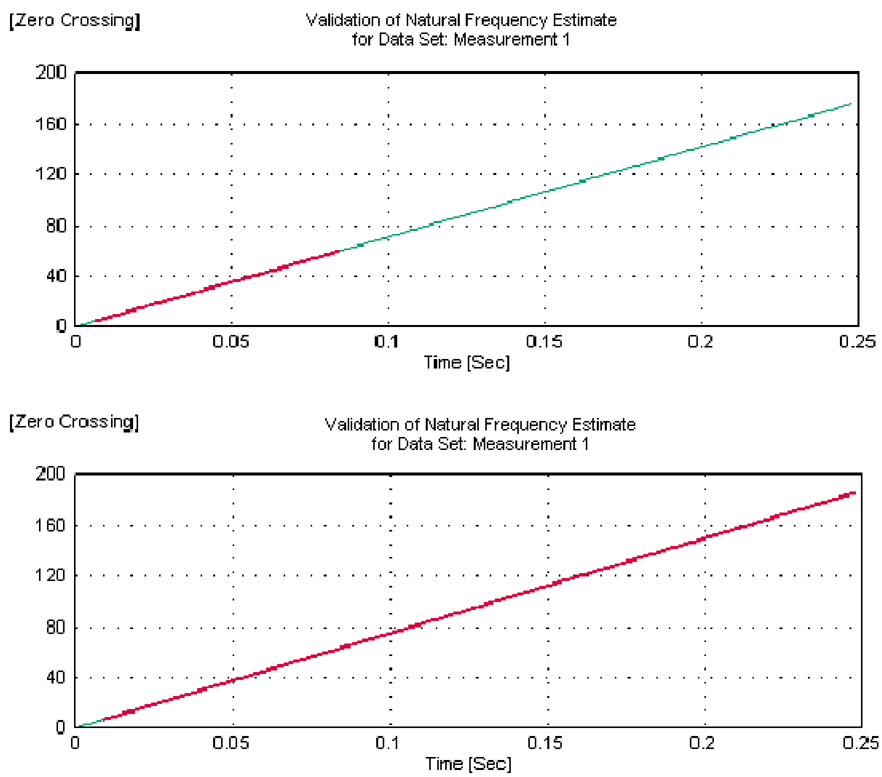

Figure 4. Natural frequency identification by zero-crossing counting without (upper) and with (lower) harmonic component. Mode at 354 Hz. Harmonic component at 374 Hz.

The number of zero-crossings as a function of time calculated from the above correlation functions is shown in Figure 4. In both cases, the zero-crossing number follows a straight line, but in the case with the harmonic component, the estimated natural frequency of the mode will be significantly biased by the presence of the harmonic component and set equal to its frequency.

In Figure 5 the damping ratio is estimated by the logarithmic decrement technique from the logarithmic envelope of the correlation function. The estimation is performed by applying a linear fit to the part of the curve being close to a straight line. Again the influence of the harmonic component is clearly visible.

Compared to the FDD technique, an improved estimate of the mode shape is obtained by using a weighted sum of the singular vectors Φ_i and singular values s_i whereby random noise is efficiently averaged out.

$$\Phi_{weight} = \sum_i \Phi_i s_i \tag{5}$$

In order not to destroy the mode shape estimate, the singular vector and singular value for the harmonic component must not be included in the summation.

In Chapter 6, the influence of harmonic components in the modal parameter identification is described quantitative by various examples.

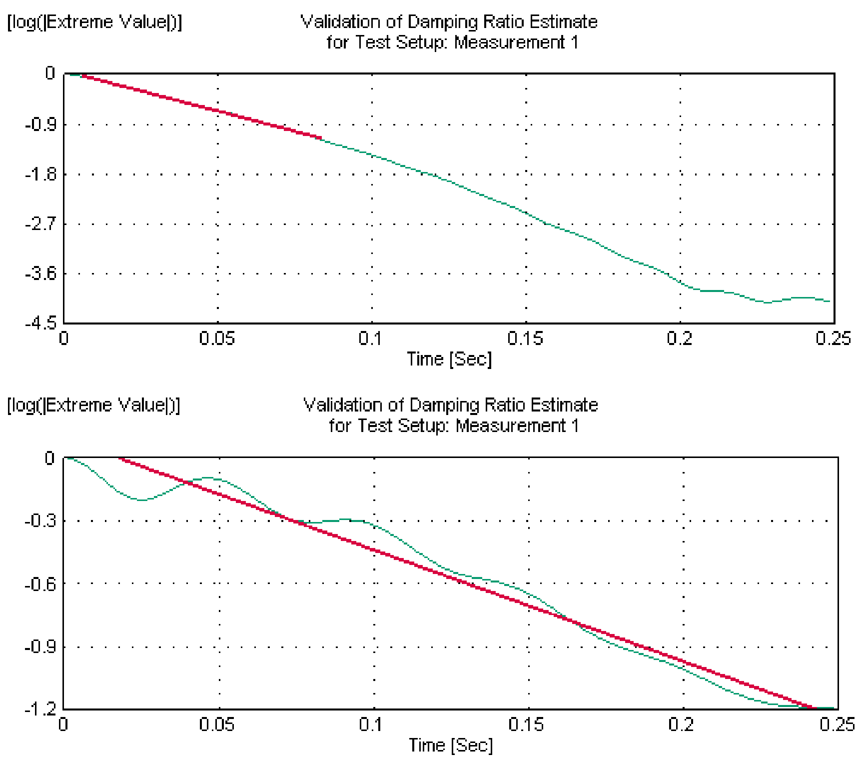

Figure 5. Damping ratio estimation from the decay of the correlation function without (upper) and with (lower) harmonic component. Mode at 354 Hz. Harmonic component at 374 Hz.

4.3 BRIEF DESCRIPTION OF THE NEW EFDD TECHNIQUE

The steps in the automated method for identifying the harmonic components are roughly as follows:

1. Each measurement channel y_i is normalized to unit variance and zero mean
2. For all frequencies f_j from DC to the Nyquist frequency a narrow bandpass filtering of y_i around f_j is performed
3. The Kurtosis $\gamma_{j,i}$ for the filtered signal y_i around f_j is calculated
4. For each frequency, the mean of the Kurtosis γ_j is calculated across the measurement channels
5. The median *m* of the Kurtosis of all frequencies is calculated. If the signal is purely Gaussian distributed this robust measure for the mean will theoretically be 0 (equation (2)).
6. For each frequency f_j the deviation of the Kurtosis γ_j from the median m is calculated. If γ_j deviates significantly from *m*, then the distribution around f_j is different than for the majority of the other frequencies. In such a case γ_j can be characterised as an outlier that should not be included in the estimation of the SDOF functions.

In Figure 6 below, the fundamental frequency at 374 Hz as well as the 2nd and 4th harmonics are automatically identified and shown as vertical green lines in the SVD plot.

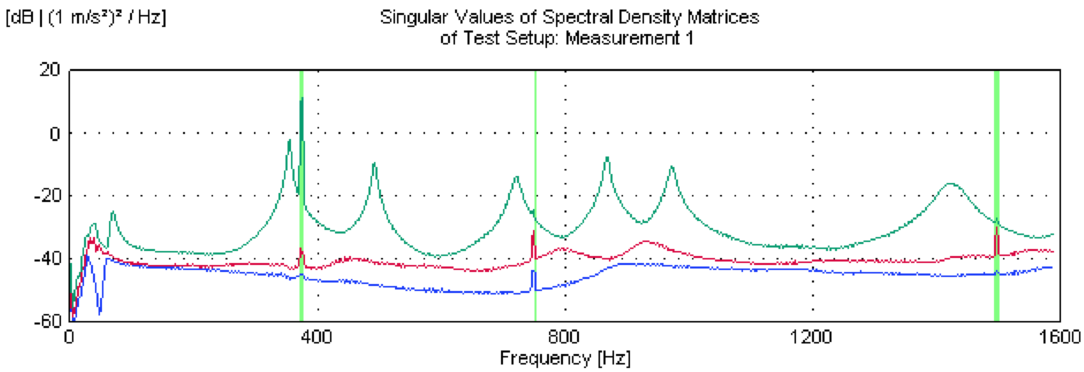

Figure 6. Indication of harmonic components at 374 Hz, 748 Hz and 1496 Hz in the SVD plot.

Knowing the frequencies of the harmonic components, the SDOF function can be estimated by removing the peaks caused by these harmonic components by using linear interpolation. The global modal parameters – natural frequency and damping – can subsequently be calculated. The local parameter – mode shape – is calculated as described in equation (5). However, only the non-interpolated singular values and vectors are used. Figure 7 shows the SDOF function in the SVD plot after removing the harmonic component.

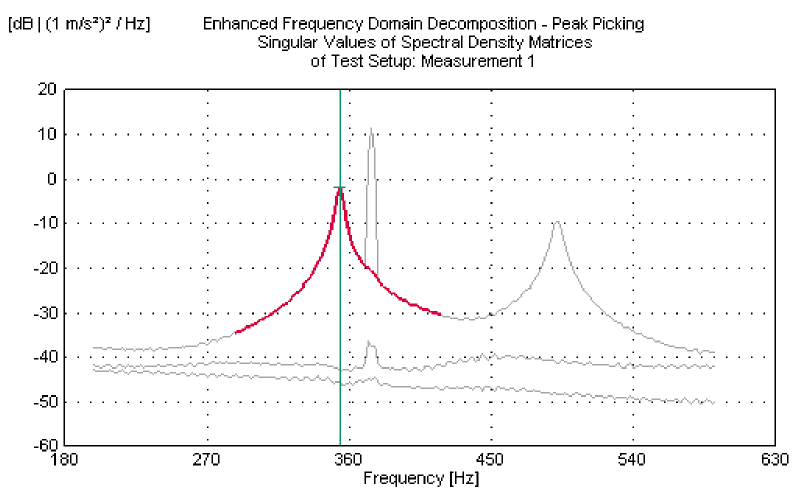

Figure 7. Removal of harmonic component in the SVD plot using linear interpolation.
Mode at 354 Hz. Harmonic component at 374 Hz.

5. MEASUREMENT SETUP

The measurements were performed using an aluminium plate structure supported by foam rubber as test object. The plate has lightly damped and well-separated modes. A Brüel & Kjær Hand-held Exciter Type 5961 was attached to provide the deterministic signal at a single fixed point. The broadband stochastic noise input was provided by finger tapping on the plate randomly in time and space to fulfil the criteria for performing OMA measurements. To avoid mass loading effects across data sets, which is not insignificant due to the low dynamic mass of the plate, all measurements were done in single data sets by using 16 accelerometers (Brüel & Kjær Type 4507-B) equally distributed over the plate. The data acquisition and analysis was performed using PULSE™ Type 3560-D front-end and a laptop PC running PULSE™ software. All measurements were done in a 1.6 kHz frequency range using 60 s of time data. A frequency resolution of 0.25 Hz and 1 Hz was used.

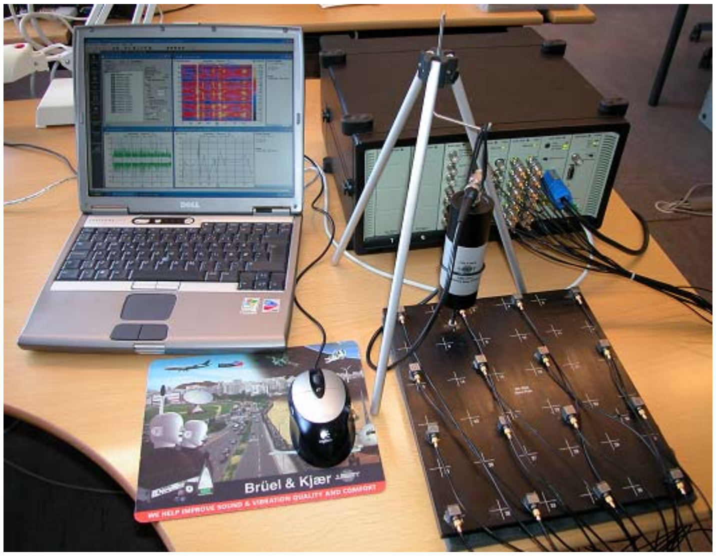

Figure 8. The measurement setup.

6. MEASUREMENT AND ANALYSIS RESULTS

Several tests were conducted to assess the robustness and accuracy of the method as shown in Table 3.

Harmonic Component [Hz]	EFDD Method				New EFDD Method			
	Natural Frequency [Hz]		Damping Ratio [%]		Natural Frequency [Hz]		Damping Ratio [%]	
	dF = 1 Hz	dF = 0.25 Hz	dF = 1 Hz	dF = 0.25 Hz	dF = 1 Hz	dF = 0.25 Hz	dF = 1 Hz	dF = 0.25 Hz
None	354	354	0.5812	0.5545	354	354	0.5812	0.5545
329	329	329	0.1237	0.03099	353.8	353.8	0.5829	0.544
334	334	334	0.1192	0.02977	353.9	353.9	0.4956	0.449
339	339	339	0.1205	0.03025	353.8	353.8	0.504	0.4687
344	344	344	0.1227	0.03038	354.1	353.8	0.4387	0.4681
349	349	349	0.131	0.03307	354.1	354.3	0.6063	0.5331
354	354	354	0.1312	0.03287	354.3	354.2	0.6387	0.5669
359	359	359	0.1268	0.03191	354.6	354.1	0.6272	0.4657
364	364	364	0.1161	0.02867	354.2	354.2	0.453	0.4093
369	369	369	0.1137	0.02897	354.4	354.3	0.4709	0.4321
374	374	374	0.1101	0.02834	354	354	0.446	0.4172
379	379	379	0.09905	0.02507	354.4	354.2	0.4652	0.4317

Table 3. Comparison of natural frequency and damping ratio using the existing EFDD and new EFFD techniques. 1st bending mode at 354 Hz.

Measurements with a single harmonic component located within the SDOF function of the plate's 1st bending mode at 354 Hz were compared to the measurement based on purely stochastic excitation. The measurements were done with a frequency resolution of 0.25 Hz and 1 Hz. The natural frequency and damping ratio were calculated for the existing EFDD technique and for the new EFDD technique with removed harmonic component.

If harmonic components are present inside the SDOF function, the existing EFDD technique will give inaccurate estimates of the modal parameters. The natural frequency will – if the peak of the harmonic component is higher than the peak of the structural mode - be estimated equal to the frequency of the harmonic component and the damping ratio will be estimated too low. The harmonic component can be seen as a forced vibration with low (theoretically zero) damping. The damping ratio consequently drops by a factor of 4, when the frequency resolution is reduced by a factor of 4 due to the reduction of the narrow-banded stochastic noise.

Using the new EFDD technique the natural frequencies and damping ratios are, in general, estimated with a good accuracy. However, when the harmonic component is close to the natural frequency, larger deviations occur. As the plate is very lightly damped, this is expected due to the use of linear interpolation. The obtained damping ratio will be higher as the calculated SDOF function gets more "flat" and the mode will consequently be estimated as more heavily damped. A polynomial fit is believed to significantly improved the calculated SDOF function and will be examined in the near future.

Also the MAC values between the actual mode shape and the estimated mode shapes obtained after removing the harmonic component give high correlation. In all cases, the MAC value was better than or equal to 0.9997.

Some of the benefits of the new EFDD method are:

- Robust method – Harmonic components are clearly identified and their effect can be eliminated even in the case of a harmonic component located exactly at a structural mode. Using high frequency resolution and/or polynomial fit is required
- No prior knowledge required – For example about the number of harmonics and their frequencies
- Easy-to-use – Automated method based on the EFDD technique
- Fast – Based on computational efficient algorithms

7. CONCLUSIONS AND FUTURE WORK

The presence of dominant harmonic components in the measured responses is unavoidable in many applications of Operational Modal Analysis. The consequences can be quite drastic, when using the Enhanced Frequency Domain Decomposition (EFDD) technique, as it requires the harmonic components to be outside the determined SDOF function. This paper has described a new method based on the EFDD technique, where the harmonic components are first identified using kurtosis and then removed by performing a linear interpolation across the harmonic components in the SDOF function.

The quality of the method has been assessed from various experiments using a plate structure excited by a combination of a single sinusoidal signal and a broadband stochastic signal. Compared to the modal results obtained with pure stochastic excitation of the same structure, the method shows good agreement in terms of natural frequency, damping ratio and mode shape. Even the effect of having a harmonic component located exactly at the peak of a structural mode can be eliminated and a good modal estimate obtained, if a high frequency resolution is used. Using a polynomial fit, instead of the simple linear interpolation used in this initial implementation, would improve the results. The method furthermore benefits from not requiring any prior knowledge of the harmonic components in terms of frequencies or levels, is computational very efficient, and really simple to use.

Future work will include examination of the method's robustness to multiple and varying frequencies located within the SDOF function. The effect of different polynomial fits will be studied as well. Finally, a new method for eliminating harmonic components before using the Stochastic Subspace Identification (SSI) techniques will be worked on.

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