Using EFDD as a Robust Technique to Deterministic Excitation in Operational Modal Analysis

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Abstract

The algorithms used in Operational Modal Analysis assume that the input forces are stochastic in nature. While this is often the case for civil engineering structures, mechanical structures, in contrast, are subject inherently to deterministic forces due to the rotating parts in the machinery. These forces can be seen as harmonic families in the responses, and their influence should if possible be eliminated before extracting the modes in their vicinity.

This paper describes a new and automated method based on the Enhanced Frequency Domain Decomposition technique. First the deterministic signals are identified from kurtosis calculations; secondly performing a linear interpolation across their presence in the identified SDOF functions significantly reduces their influence. No prior knowledge of the deterministic signals is required.

For assessing the quality of the method, various experiments were carried out on a plate structure excited by respectively a pure stochastic signal and the same stochastic signal superimposed by a deterministic signal. Good agreement was found in terms of both natural frequencies, damping ratios and mode shapes. Even the influence of a deterministic signal located exactly at a natural frequency can be practically eliminated.

1 Introduction to Operational Modal Analysis

Operational Modal Analysis (OMA) is based on measuring the output of a structure only and using the ambient and natural operating forces as unmeasured input. It is used instead of classical mobility-based modal analysis for accurate modal identification under actual operating conditions, and in situations where it is difficult or impossible to artificially excite the structure.

The algorithms used in OMA assume that the input forces are stochastic in nature. This is often the case for civil engineering structures like buildings, towers, bridges and offshore structures, which are mainly loaded by ambient forces like wind, waves, traffic or seismic micro-tremors. The loading forces of many mechanical structures are, however, often more complex. They are typically a combination of deterministic signals originating from the rotating and reciprocating parts and broadband excitation originating from either self-generated vibration from, for example, bearings and combustions or from ambient excitations like air turbulence and road vibrations. However, civil engineering structures can also have broadband responses superimposed by deterministic signals from, for example, ventilation systems, turbines and generators.

As the input forces to the structure are not measured in OMA, attention must be paid to identify the deterministic signals and reduce their influence in the modal parameter extraction process.

This paper starts by describing the consequences of having deterministic signals present in the responses for different modal parameter identification techniques. Then a brief overview of various methods for separating deterministic signals from modes is given, before explaining in more depth a very easy-to-use and robust method based on kurtosis. The well-known and popular Enhanced Frequency Domain Decomposition modal identification technique is briefly explained, leading to the description of an improved technique for reducing the influence of deterministic signals in OMA.

Finally the quality of the method is assessed from various experiments using a plate structure. The structure is excited by a combination of a single sinusoidal signal and a broadband stochastic signal and compared to the results obtained with pure stochastic excitation of the same structure.

2 Consequences of Deterministic Signals

The consequences of having deterministic signals present in the responses depend on the nature of the deterministic signals (number, frequency and level) and the modal parameter extraction method used. Table 1 below indicates these consequences for the Frequency Domain Decomposition (FDD), Enhanced FDD (EFDD) and Stochastic Subspace Identification (SSI) techniques. See [1], [2] for further description of the FDD, EFDD and SSI techniques. The EFDD method is briefly explained in Chapter 4.1 as well.

Techniques	**Consequences**
All techniques	• Deterministic signals are potentially mistaken for being modes • Deterministic signals might potentially bias the estimation of the modes (natural frequency, modal damping, mode shape) • A high dynamic range might be required to extract “weak” modes
FDD	• The picked FFT line might be biased by the deterministic signal(s) • Deterministic signals must be away from the modes (only the picked FFT line is used in the FDD technique)
EFDD	• The identified SDOF function used for modal parameter estimation might be biased by the deterministic signal(s) • Deterministic signals must be outside the SDOF function thereby potentially narrowing the SDOF function and resulting in poorer identification (leakage)
SSI (PC, UPC, CVA)	• The SSI methods will (theoretically) estimate both deterministic signals and modes - even for deterministic signals close to - or with equal frequencies as - the modes • Information in the time signal is used both to extract the deterministic signals and the modes, therefore the recording time should be longer • Significantly higher model orders are required to separate deterministic signals from modes

Table 1. Consequences of deterministic signals for various identification techniques.

For the EFDD technique it is important that deterministic signals inside the SDOF functions are identified and their influence significantly reduced before extracting the modal parameters.

It should be noticed that deterministic signals cannot, in general, be removed by simple filtering as this would in most practical cases significantly change the poles of the modes and thereby their natural frequency and modal damping.

3 Identification of Deterministic Signals

In [3] various methods for identifying deterministic signals and structural modes were investigated. This included: Short Time Fourier Transform, Singular Value Decomposition, Visual Mode Shapes Comparison, Modal Assurance Criterion, Stabilization Diagram and Probability Density Functions.

In addition, kurtosis can also be used as a deterministic signal indicator. The kurtosis γ of a stochastic variable x provides a way of expressing how peaked or how flat the probability density function of x is. The kurtosis is defined as the fourth central moment of the stochastic variable normalised with respect to the standard deviation σ as follows, where μ is the mean value of x and E is denoting the expectation value [4]:

$$\gamma(x|\mu,\sigma) = \frac{E\left[(x-\mu)^4\right]}{\sigma^4} \tag{1}$$

Subtracting 3 from equation (1) gives a kurtosis of zero, when x is normally distributed [5]:

$$\gamma^*(x|\mu,\sigma) = \frac{E\left[(x-\mu)^4\right]}{\sigma^4} - 3 \tag{2}$$

Using equation (2), a PDF with a positive kurtosis is said to be leptokurtic. If the kurtosis is negative, it is said to be platykurtic. A PDF with kurtosis equal to zero is called mesokurtic. Leptokurtosis is associated with PDFs that are simultaneously “peaked” and have “fat tails.” Platykurtosis is associated with PDFs that are simultaneously less peaked and have thinner tails. The PDF of the response of a pure mode will be normally distributed, and hence the kurtosis $\gamma^* = 0$ (mesokurtic). Its PDF y is given by:

$$y = f(x|\mu,\sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}} \tag{3}$$

A deterministic signal can be treated as a stochastic signal if the phase varies randomly within its period. In the case of a pure sinusoidal signal, the PDF will have two distinct peaks approaching infinity at ±a, where a is the amplitude of the deterministic signal. The PDF is given by:

$$y = f(x|a) = \left(\pi\cos(\arcsin(x/a))\right)^{-1} \tag{4}$$

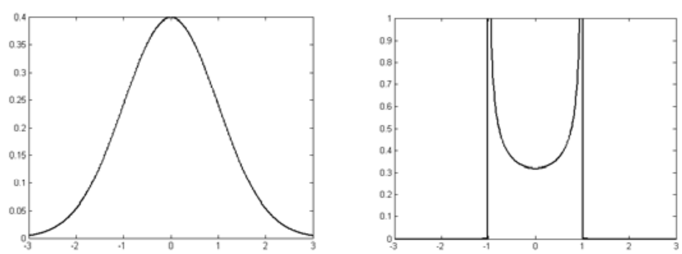

Figure 1. Normalized PDF of the response of a mode (left) and a sinusoidal signal (right).

The kurtosis $\gamma^* = -1½$ for a sinusoidal signal. The difference in kurtosis of various signals is used in the deterministic signal detection technique described in this paper.

4 Reducing the Influence of Deterministic Signals

4.1 Brief Description of the Existing EFDD Technique

The EFDD technique is an extension to the FDD technique. FDD is a basic technique that is extremely easy to use. You simply pick the modes by locating the picks in SVD plots calculated from the spectral density spectra of the responses. Animation is performed immediately. As the FDD technique is based on using a single frequency line from the FFT analysis, the accuracy of the estimated natural frequency depends on the FFT resolution and no modal damping is calculated. Compared to FDD, the EFDD gives an improved estimate of both the natural frequencies and the mode shapes and also includes damping.

In EFDD, the SDOF Power Spectral Density function, identified around a resonance peak, is taken back to the time domain using the Inverse Discrete Fourier Transform (IDFT). The natural frequency is obtained by determining the number of zero-crossing as a function of time, and the damping by the logarithmic decrement of the corresponding SDOF normalized auto correlation function. The SDOF function is estimated using the shape determined by the previous FDD peak picking - the latter being used as a reference vector in a correlation analysis based on the Modal Assurance Criterion (MAC). A MAC value is computed between the reference FDD vector and a singular vector for each particular frequency line. If the MAC value of this vector is above a user-specified MAC Rejection Level, the corresponding singular value is included in the description of the SDOF function. The lower this MAC Rejection Level is, the larger the number of singular values included in the identification of the SDOF function will be.

In the left-hand side of Figure 2, the estimated SDOF function for the plate's first bending mode is shown. A pure SDOF function can be extracted. In the right-hand side of Figure 2, a deterministic signal close to the resonance frequency is present. As seen, the SDOF function is severely distorted by the presence of the deterministic signal and hence the estimation of the modal parameters will be incorrect.

Performing an IDFT on the above SDOF functions the normalized correlation functions are calculated. In the left-hand side of Figure 3, a typical response is seen of a resonating system that decays exponentially. In the right-hand side of Figure 3, the effect of the deterministic signal is clearly visible. The deterministic signal can be thought of as a forced vibration with very low damping. The decay is significantly longer and beating phenomena are observed.

The number of zero-crossings as a function of time calculated from the above correlation functions is shown in Figure 4. In both cases, the zero-crossing number follows a straight line, but in the case with the deterministic signal, the estimated natural frequency of the mode will be significantly biased by the presence of the deterministic signal and set equal to its frequency.

In Figure 5 the damping ratio is estimated by the logarithmic decrement technique from the logarithmic envelope of the correlation function. The estimation is performed by applying a linear fit to the part of the curve being close to a straight line. Again the influence of the deterministic signal is clearly visible.

Compared to the FDD technique, an improved estimate of the mode shape Φ_{weight} is obtained by using a weighted sum of the singular vectors Φ_i and singular values s_i whereby random noise is efficiently averaged out. In order not to destroy the mode shape estimate, the singular vector and singular value for a deterministic signal must not be included in the summation.

$$\Phi_{weight} = \sum_i \Phi_i s_i \tag{5}$$

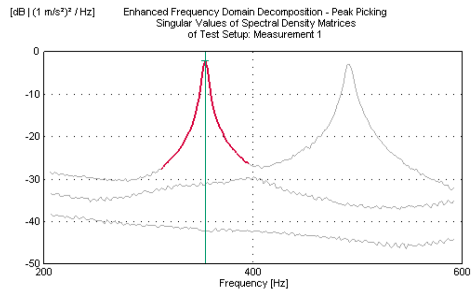

Figure 2. Singular Value SDOF identification without (left) and with (right) deterministic signal. Mode at 354 Hz. Deterministic signal at 374 Hz.

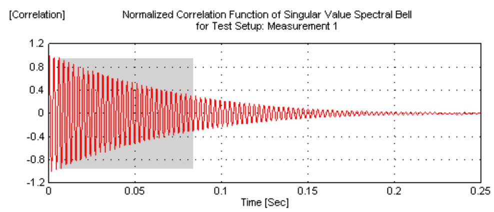

Figure 3. Normalized correlation function without (left) and with (right) deterministic signal. Mode at 354 Hz. Deterministic signal at 374 Hz. Grey area indicates the part of the function used.

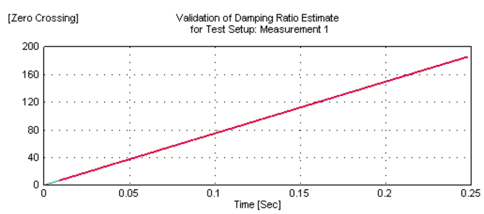

Figure 4. Natural frequency identification by zero-crossing counting without (left) and with (right) deterministic signal. Mode at 354 Hz. Deterministic signal at 374 Hz.

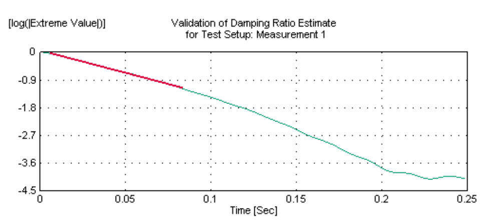

Figure 5. Damping ratio estimation from the decay of the correlation function without (left) and with (right) deterministic signal. Mode at 354 Hz. Deterministic signal at 374 Hz.

4.2 Brief Description of the New EFDD Technique

The steps in the automated method for identifying the deterministic signals are roughly as follows:

1. Each measurement channel y_i is normalized to unit variance and zero mean
2. For all frequencies f_j a narrow bandpass filtering of y_i around f_j is performed
3. The Kurtosis $\gamma_{j,i}$ for the filtered signal y_i around f_j is calculated
4. For each frequency, the mean of the Kurtosis γ_j is calculated across the measurement channels
5. The median m of the Kurtosis of all frequencies is calculated. If the signal is purely Gaussian distributed this robust measure for the mean will theoretically be 0 (equation (2))
6. For each frequency f_j the deviation of the Kurtosis γ_j from the median m is calculated. If γ_j deviates significantly from m, the distribution around f_j is different than for the majority of the frequencies. Hence γ_j is an outlier and should be excluded from the SDOF functions estimation

In the left-hand side of Figure 6, the fundamental frequency at 374 Hz as well as the 2nd and 4th harmonics are automatically identified and shown as vertical green lines in the SVD plot. Knowing the frequencies of the deterministic signals, the SDOF function can be estimated by removing the peaks caused by these deterministic signals by using linear interpolation. The global modal parameters – natural frequency and damping – can subsequently be calculated. The local parameter – mode shape – is calculated as described in equation (5). However, only the non-interpolated singular values and vectors are included. The right-hand side of Figure 6 shows the SDOF function in the SVD plot after removing the deterministic signal.

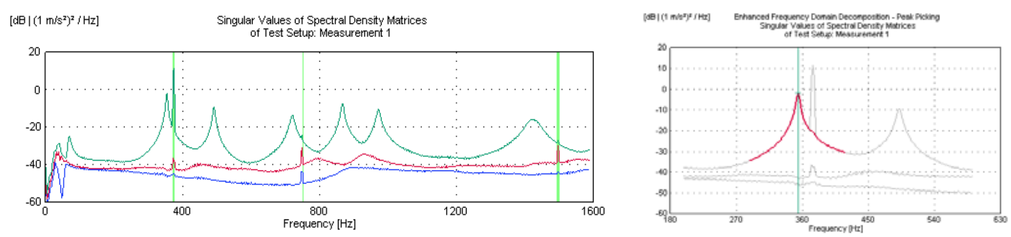

Figure 6. SVD plots. Identification of deterministic signals at 374 Hz, 748 Hz and 1496 Hz (left). Removal of deterministic signal at 374 Hz around mode at 354 Hz using linear interpolation (right).

5 Measurement Setup

The measurements were performed using an aluminium plate structure supported by foam rubber as test object. The plate has lightly damped and well-separated modes. A Brüel & Kjær Hand-held Exciter Type 5961 was attached to provide the deterministic signal at a single fixed point. The broadband stochastic noise input was provided by finger tapping on the plate randomly in time and space to fulfil the requirements for performing OMA tests.

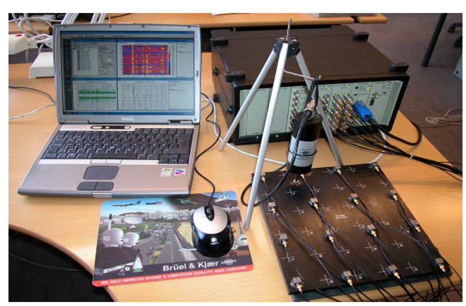

Figure 7. Measurement setup.

To reduce mass loading effects, all measurements were done in single data sets using 16 Brüel & Kjær Type 4507-B accelerometers equally distributed over the plate. Data acquisition and analysis was done using a PULSE™ Type 3560-D front-end and a PC running PULSE™ software. All measurements were done in a 1.6 kHz frequency range using 60 s of time data. A frequency resolution of 0.25 Hz and 1 Hz was used.

6 Measurement and Analysis Results

Several tests were conducted to assess the robustness and accuracy of the method as shown in Table 2 below. Measurements with a single deterministic signal located within the SDOF function of the plate's 1st bending mode at 354 Hz were compared to the measurement based on purely stochastic excitation. The measurements were done with a frequency resolution of 0.25 Hz and 1 Hz. The natural frequency and damping ratio were calculated for the existing EFDD technique and for the new EFDD technique with removed deterministic signal.

If deterministic signals are present inside the SDOF function, the existing EFDD technique will give inaccurate estimates of the modal parameters. The natural frequency will - if the peak of the deterministic signal is higher than the peak of the mode - be estimated equal to the frequency of the deterministic signal and the damping ratio will be estimated too low. The deterministic signal can be seen as a forced vibration with low (theoretically zero) damping. The damping ratio consequently drops by a factor of 4, when the frequency resolution is reduced by a factor of 4 due to the reduction of the narrow-banded stochastic noise.

Deterministic Signal [Hz]	EFDD Method				New EFDD Method			
	Natural Frequency [Hz]		Damping Ratio [%]		Natural Frequency [Hz]		Damping Ratio [%]	
	dF = 1 Hz	dF = 0.25 Hz	dF = 1 Hz	dF = 0.25 Hz	dF = 1 Hz	dF = 0.25 Hz	dF = 1 Hz	dF = 0.25 Hz
None	354	354	0.581	0.554	354	354	0.581	0.554
329	329	329	0.124	0.0310	353.8	353.8	0.583	0.544
334	334	334	0.119	0.0298	353.9	353.9	0.496	0.449
339	339	339	0.121	0.0303	353.8	353.8	0.504	0.469
344	344	344	0.123	0.0304	354.1	353.8	0.439	0.468
349	349	349	0.131	0.0331	354.1	354.3	0.606	0.533
354	354	354	0.131	0.0329	354.3	354.2	0.639	0.567
359	359	359	0.127	0.0319	354.6	354.1	0.627	0.466
364	364	364	0.116	0.0287	354.2	354.2	0.453	0.409
369	369	369	0.114	0.0290	354.4	354.3	0.471	0.432
374	374	374	0.110	0.0283	354.0	354.0	0.446	0.417
379	379	379	0.099	0.0251	354.4	354.2	0.465	0.432

Table 2. Comparison of natural frequencies and damping ratios using the existing EFDD and new EFFD techniques. 1st bending mode at 354 Hz.

Using the new EFDD technique the natural frequencies and damping ratios are, in general, estimated with a good accuracy. However, when the deterministic signal is close to the natural frequency, larger deviations occur. As the plate is very lightly damped, this is expected due to the use of linear interpolation. The obtained damping ratio will be higher as the calculated SDOF function gets more "flat" and the mode will consequently be estimated as more heavily damped. A polynomial fit is believed to significantly improve the calculated SDOF function and will be examined in the near future.

Also the MAC values between the actual mode shape and the estimated mode shapes obtained after removing the deterministic signal give high correlation. In all cases, the MAC value was better than or equal to 0.9997.

The benefits of the new EFDD method include:

- Robustness - Deterministic signals are clearly identified and their effect significantly reduced even when they are located exactly at a natural frequency. High frequency resolution and/or polynomial fit is required
- No prior knowledge required - E.g. the number of deterministic signals and their frequencies
- Ease-of-use - Automated method based on the EFDD technique

7 Conclusions and Future Work

The presence of dominant deterministic signals in the measured responses is unavoidable in many applications of operational modal analysis. The consequences can be quite drastic, when using the EFDD technique, as the deterministic signals must be outside the determined SDOF functions. This paper has described a new method based on the EFDD technique, where the deterministic signals are first identified using kurtosis and then removed by performing linear interpolation across the deterministic signals in the SDOF functions.

The quality of the method has been assessed from various experiments using a plate structure excited by a combination of a sinusoidal signal and a broadband stochastic signal. Compared to the modal results obtained with pure stochastic excitation of the same structure, the method shows good agreement in terms of natural frequency, damping ratio and mode shape. Even the effect of having a sinusoidal signal located exactly at a natural frequency can be significantly reduced and a good modal estimate obtained, if high frequency resolution is used. Using a polynomial fit, instead of the simple linear interpolation used in this initial implementation, would improve the results further. The method furthermore benefits from not requiring any prior knowledge of the deterministic signals in terms of frequencies or levels and is really simple to use.

Future work will include examination of the method's robustness to multiple and varying frequencies located within the SDOF function. The effect of different polynomial fits will be studied as well. Finally, a new method for eliminating deterministic signals before using the Stochastic Subspace Identification techniques will be worked on.

8 References

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