

# STATISTICAL DAMAGE DETECTION OF CIVIL ENGINEERING STRUCTURES USING ARMAV MODELS

*P. Andersen & P.H. Kirkegaard*

*Aalborg University*

*Department of Building Technology and Structural Engineering*

*Sohngaardsholmsvej 57, DK-9000 Aalborg, Denmark*

## ABSTRACT

In this paper a statistically based damage detection of a lattice steel mast is performed. By estimation of the modal parameters and their uncertainties it is possible to detect whether some of the modal parameters have changed with a statistical significance. The estimation of the uncertainties is based on ARMAV calibration using a non-linear Prediction Error Method approach. Besides estimating the parameters of the ARMAV model this approach can also provide an estimate of the covariance matrix of these parameters. On the basis of this covariance matrix it is possible to estimate the uncertainties of the modal parameters.

## NOMENCLATURE

$A_i$	Auto-regressive coefficient matrix
$C_i$	Moving average coefficient matrix
$F$	State matrix
$B$	Input matrix
$C$	Observation matrix
$A$	Transition matrix
$T$	Sampling interval
$y(t)$	Continuous-time system response
$y(t_k)$	Discrete-time system response
$w(t)$	Continuous-time Gaussian white noise input
$e(t_k)$	Discrete-time innovation process
$\hat{\theta}_N$	Parameter vector based on $N$ samples
$\hat{P}_\theta$	Estimated covariance matrix of $\hat{\theta}_N$
$\psi_j$	$j$ th complex eigenvector
$\Phi_j$	$j$ th complex mode shapes
$f_i$	$i$ th natural eigenfrequency
$\zeta_i$	$i$ th damping ratio

## 1. INTRODUCTION

It is a well-known fact that estimated modal parameters of a structural system can serve to indicate for whether damage of a structure has occurred or not, see Rytter [10]. Changes in natural eigenfrequencies are no doubt the most frequently used damage indicators. These are sensitive to both local and global damages. A local damage will cause changes in the derivatives of the mode shapes at the position of the damage. This means that a mode shape having many coordinates can locate the approximate position of a damage. The introduction of damage in a structure will usually cause changes in the damping capacity of the structure. Damping ratios can therefore be sensitive to the introduction of even small cracks in a structure.

Thus, all modal parameters are in principle applicable as damage indicators, see Rytter [10]. This means that they can be used at least for detection of damage. However, the key to a successful damage detection is the use of unbiased and low-variance modal parameter estimates as damage indicators. If the estimates are biased they might cause a false alarm, i.e. indicate a damage that does not exist. If the estimation inaccuracies are too dominant, it might be impossible to detect any significant changes, i.e., the existence of a damage might be hidden. Thus, if the uncertainties of the estimated modal parameters can be estimated it will be possible to assess whether changes of modal parameters are caused by e.g. a damage or simply by estimation uncertainties. Further, if the changes of the modal parameters not are caused by estimation inaccuracies, the estimated uncertainties can be used to establish a probabilistic confidence in the existence of a damage, see Doebling et al. [6] and Kirkegaard et al. [8].

In this paper, statistically based damage detection is applied to ambient excited civil engineering structures. It is shown that an ambient excited civil engineering structure can be represented by an Auto-Regressive Moving Average Vector (ARMAV) model. The applied system identification method is a non-linear Prediction Error Method (PEM). Estimation of ARMAV models using this technique is known to provide asymptotically unbiased and efficient modal parameter estimates solely on the basis of output measurements of a system. In other words, the uncertainties of the estimated modal parameters will attain the Cramer-Rao lower bound of variance. By assuming that the lower bound is reached, it is possible to estimate the standard deviations associated with the estimated modal parameters. It is shown how to apply this additional information as a basis for a simple statistically based damage detection, which will be illustrated on measurements of a lattice steel mast that has been damaged.

In section 2, it is explained how an ambient excited structure can be modelled by an ARMAV model. The estimation of the ARMAV model is presented in section 3. From the estimated ARMAV model the modal parameters and their uncertainties can be estimated. This is shown in section 4. In section 5, an example of statistical damage detection will be given. The technique will be illustrated on the basis of measurements of a lattice steel mast. Finally, in section 6 conclusions will be made.

## 2. ARMAV MODELLING OF AMBIENT EXCITED STRUCTURES

Experience has led to the following mathematical mass-spring-dashpot lumped parameter model for a structure subjected to external loading

$$M\ddot{y}(t) + C\dot{y}(t) + Ky(t) = f(t) \quad (1)$$

$M$ ,  $C$  and  $K$  are the mass, damping and stiffness matrices all of dimensions  $p \times p$ .  $y(t)$  and  $f(t)$  is the  $p \times 1$  displacement and  $p \times 1$  force vectors at the mass points, respectively. From a system identification point of view a generalization of the mathematical model is necessary, since the number of measurement channels is usually less than the number of identified modes. A generalized multivariate model can be formulated as, Andersen [5]

$$D^s y(t) + A_{y,s-1} D^{s-1} y(t) + \dots + A_{y,0} y(t) = B_{f,s-2} D^{s-2} f(t) + \dots + B_{f,0} f(t) \quad (2)$$

where  $D$  is a differential operator. The matrices  $A_{y,i}$  and  $B_{f,i}$  are all of the dimension  $p \times p$ . The displacement vector  $y(t)$  and its derivatives are all of the dimension  $p \times 1$ . The  $p \times 1$  vector  $f(t)$  describes the forces applied to the system. The modes of a structural system will typically be underdamped, which implies that each mode is described by a pair of complex conjugated eigenvalues. In this situation the order  $s$  will be defined as  $s = \frac{2N}{p}$  with  $N$  being the number of underdamped modes.

It is often assumed that the ambient excitation  $f(t)$  is given as the output of a linear time-invariant shaping filter subjected to Gaussian white noise. Due to the Gaussian assumption, it is implicitly assumed that the true ambient excitation is at least weakly stationary. If the ambient excitation can be described by filtered white noise, it is possible to derive a model for it. Assume that the excitation  $f(t)$  of the structural system is obtained as the output of an  $m$ th-order  $p$ -variate linear time-invariant continuous-time shaping filter

$$D^m f(t) + A_{f,m-1} D^{m-1} f(t) + \dots + A_{f,0} f(t) = w(t) \quad (3)$$

For simplicity, it is assumed that  $f(t)$  and  $w(t)$  have the same dimensions as  $z(t)$ . This implies that the matrices  $A_{f,i}$  all have the dimension  $p \times p$ . The stochastic process  $w(t)$  is a zero-mean Gaussian white noise, fully described by its covariance function. This covariance function is defined in terms of the  $p \times p$  intensity matrix  $W$  as

$$E[w(t)] = 0, \quad E[w(t)w^T(t-\tau)] = \delta(\tau)W \quad (4)$$

where  $\delta(\tau)$  is the Dirac delta function. These statistical properties are abbreviated  $NID(0, W)$ .

It is obvious that the response  $y(t)$  of the system will contain a mixture of the dynamic behaviour of the structural system and of the excitation. It is also intuitively clear that during a system identification the dynamic modes of the shaping filter will also be estimated. These modes are, together with any noise modes, called non-physical modes. In this way they can be distinguished from the physical modes of the structural system.

The structural system can then be combined with the shaping filter of the excitation by means of convolution into a resulting linear system subjected to a Gaussian white noise. The resulting differential equation system will be of the order  $n = s+m$ . Such a differential equation system can be represented by the following state space system, Andersen [5]

$$\begin{aligned} \dot{x}(t) &= Fx(t) + Bw(t), \quad w(t) \in NID(0, W) \\ y(t) &= Cx(t) \end{aligned} \quad (5)$$

where  $F$  is the  $np \times np$  state matrix,  $B$  is the  $np \times p$  input matrix, and  $C$  the  $p \times np$  observation matrix. Define a discrete time instance as  $t_k = kT$ , where  $k$  is an integer and  $T$  is the sampling interval. A sampling of the solution of (5) then leads to

$$\begin{aligned} x(t_{k+1}) &= Ax(t_k) + \tilde{w}(t_k), \quad \tilde{w}(t_k) \in NID(0, \Omega) \\ y(t_k) &= Cx(t_k) \end{aligned} \quad (6)$$

where the process  $\tilde{w}(t_k)$  is a discrete-time Gaussian white noise that is completely described by the covariance matrix  $\Omega$ , given by

$$\Omega = \int_0^T e^{F' t} B W B^T e^{F t} dt \quad (7)$$

The transition matrix  $A$  is defined as

$$A = e^{FT} \quad (8)$$

whereas  $C$  is unaffected by the sampling. In Andersen et al. [2] and Andersen [5], it is shown how the state space system (5) can be represented in discrete time by a covariance equivalent  $p$ -variate ARMAV( $n, n-1$ ) model, i.e. an ARMAV model with an  $n$ th order autoregressive part and a moving average part of order  $n-1$ .

However, (6) does not account for the presence of noise which will most certainly always be present. A way to incorporate a noise description into the ARMAV model is to add process and measurement noise to the sampled state space system, Andersen [3] and [5]

$$\begin{aligned} x(t_{k+1}) &= Ax(t_k) + \tilde{w}(t_k) + w(t_k) \\ y(t_k) &= Cx(t_k) + v(t_k) \end{aligned} \quad (9)$$

These noise terms are all assumed zero-mean with a joint second-order moment given by

$$E \begin{bmatrix} \tilde{w}(t) \\ w(t) \\ v(t) \end{bmatrix} \begin{bmatrix} \tilde{w}^T(t) & w^T(t) & v^T(t) \end{bmatrix} = \begin{bmatrix} \Omega & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & Q & S^T \\ \mathbf{0} & S & R \end{bmatrix} \quad (10)$$

Since external noise is now present in the system the system response at a given time step cannot be calculated explicitly, but only predicted. This prediction is performed by the means of a Kalman filter. From this filter a  $p$ -variate ARMAV( $n,n$ ) model that describes the system dynamics as well as the noise, can be derived, Andersen [5].

$$y(t_k) + A_1 y(t_{k-1}) + \dots + A_n y(t_{k-n}) = e(t_k) + C_1 e(t_{k-1}) + \dots + C_n e(t_{k-n}) \quad (11)$$

$$e(t_k) \in NID(\mathbf{0}, \Lambda)$$

The left-hand side of this difference equation system is the auto-regressive part that describes the system dynamics. The left-hand side is the moving average part that describes the external noise as well as the white noise excitation, and secures stationarity of the system response.  $e(t_k)$  is a stationary zero-mean Gaussian white noise innovation process, described by the covariance matrix  $\Lambda$ . The matrices  $A_i$  and  $C_i$  are the auto-regressive and the moving average coefficient matrices, respectively. The auto-regressive coefficient matrices are obtained as

$$\begin{bmatrix} A_n & A_{n-1} & \dots & A_1 \end{bmatrix} = -CA^n \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}^{-1} \quad (12)$$

This follows directly from the following relation that links the auto-regressive coefficient matrices to the state space matrices, Andersen [5]

$$CA^n + A_1 CA^{n-1} + \dots + A_{n-1} CA + A_n C = \mathbf{0} \quad (13)$$

The conversion from the ARMAV model back to state space is not unique. Several ways to realise the model in state space exist. These realizations can e.g. be balanced or canonical forms, Andersen [5].

### 3. ESTIMATION OF ARMAV MODELS USING THE PREDICTION ERROR METHOD

The parameter estimates, based on  $N$  samples, and returned in  $\hat{\theta}_N$  can be obtained as the global minimum point of the criterion function

$$V_N(\theta) = \det \left( \frac{1}{N} \sum_{k=1}^N \varepsilon(t_k, \theta) \varepsilon^T(t_k, \theta) \right) \quad (14)$$

In other words, as  $\hat{\theta}_N = \arg \min_{\theta} V_N(\theta)$ .

The model parameter vector  $\theta$  is determined so that the prediction error, defined as

$$\varepsilon(t_k, \theta) = y(t_k) - \hat{y}(t_k | t_{k-1}; \theta) \quad (15)$$

is as small as possible.  $\hat{y}(t_k | t_{k-1}; \theta)$  is the one-step ahead predicted system response. The  $m \times 1$  parameter vector  $\theta$  is organised in the following way

$$\theta = \text{col} \left( \begin{bmatrix} A_1 & \dots & A_n & C_1 & C_n \end{bmatrix} \right) \quad (16)$$

where *col* means stacking of all columns of the argument matrix. The total number of adjustable parameters in  $\theta$  is as such  $m = 2np^2$ . The predictor of the ARMAV( $n,n$ ) model is defined as

$$\begin{aligned} \hat{y}(t_k | t_{k-1}; \theta) = & \\ & -A_1(\theta)y(t_{k-1}) - \dots - A_n(\theta)y(t_{k-n}) + \\ & C_1(\theta)\varepsilon(t_{k-1}, \theta) + \dots + C_n(\theta)\varepsilon(t_{k-n}, \theta) \end{aligned} \quad (17)$$

This relation reveals that the predictor of the ARMAV model is non-linear, since the prediction errors themselves depend on the parameter vector  $\theta$ . This implies that an iterative minimization procedure such as the following Gauss-Newton search scheme has to be applied.

$$\begin{aligned} \hat{\theta}_N^{i+1} &= \hat{\theta}_N^i + \mu_i R_N^{-1}(\hat{\theta}_N^i) F_N^i(\hat{\theta}_N^i) \\ R_N(\theta) &= \sum_{k=1}^N \Psi(t_k, \theta) Q_N^{-1}(\theta) \Psi^T(t_k, \theta) \\ F_N(\theta) &= \sum_{k=1}^N \Psi(t_k, \theta) Q_N^{-1}(\theta) \varepsilon(t_k, \theta) \\ Q_N(\theta) &= \frac{1}{N} \sum_{k=1}^N \varepsilon(t_k, \theta) \varepsilon^T(t_k, \theta) \end{aligned} \quad (18)$$

The dimensions of  $R_N(\theta)$  and  $F_N(\theta)$  are  $m \times m$  and  $m \times 1$ , respectively.  $\mu_i$  is a bisection constant that adjusts the step size.  $\Psi(t_k, \theta)$  is the gradient of the predictor (17), i.e. the derivative of (17) with respect to each of the adjustable parameters of the ARMAV model. At each time step this gradient forms an  $m \times p$  dimensional matrix.

The estimate of the parameters of the ARMAV model can as such be calculated by supplying an initial parameter estimate. On the basis of this the prediction errors can be calculated, the matrix  $R_N(\theta)$  and the vector  $F_N(\theta)$  can be calculated. An updated estimate can then be calculated using (17). This method is called the prediction error method (PEM) since it is the prediction errors that are minimized, see Ljung [9].

For Gaussian distributed prediction errors this method is asymptotically efficient. In this case an estimate of the uncertainties of the estimate is provided by the covariance matrix, Andersen [3]

$$\hat{P}(\hat{\theta}_N) = R_N^{-1}(\hat{\theta}_N) \quad (19)$$

When the amount of data is limited the estimator will not be efficient. However, the performance of the PEM can in this situation be improved by a backward forecasting approach, Andersen et al. [1].

#### 4. EXTRACTING MODAL PARAMETERS AND ESTIMATION OF THEIR UNCERTAINTIES

The free vibrations of an ARMAV model realised in state space are described by the deterministic part of (8) as

$$\begin{aligned} x(t_{k+1}) &= Ax(t_k) \\ y(t_k) &= Cx(t_k) \end{aligned} \quad (20)$$

The solution of this system is assumed to be of the form  $x(t_k) = \psi\mu^k$ , where  $\psi$  is an  $np \times 1$  complex vector and  $\mu$  is a complex constant. Insertion into (19) yields

$$\begin{aligned} \psi\mu^{k+1} &= A\psi\mu^k \\ y(t_k) &= C\psi\mu^k \end{aligned} \quad (21)$$

showing that  $x(t_k) = \psi\mu^k$  only is a solution if and only if  $\psi$  is a solution to the first-order eigenvalue problem

$$(I\mu - A)\psi = 0 \quad (22)$$

This eigenvalue problem only has non-trivial solutions if its characteristic polynomial is satisfied. The order of this real-valued polynomial is  $np$ . Thus, there will be  $np$  roots  $\mu_j$  that are the eigenvalues of  $A$ . For each of these eigenvalues there is a non-trivial solution vector  $\psi_j$  which is the corresponding eigenvector. The mode shape  $\Phi_j$  is then obtained from (20) as

$$\Phi_j = C\psi_j, \quad j=1, 2, \dots, np \quad (23)$$

The continuous-time eigenvalues  $\lambda_j$ , the natural eigenfrequencies  $f_j$ , and damping ratios  $\zeta_j$ , can be extracted from the discrete-time eigenvalues as

$$\left. \begin{aligned} \lambda_j &= \frac{\log(\mu_j)}{T} \\ f_j &= \frac{|\lambda_j|}{2\pi} \\ \zeta_j &= -\frac{\text{Re}(\lambda_j)}{|\lambda_j|} \end{aligned} \right\} j=1, 2, \dots, np \quad (24)$$

Due to the relation (13) these modal parameters are also the modal parameters of the ARMAV model. Above, it was established that the PEM estimator for Gaussian distributed prediction errors would be statistically efficient. A standard for the estimation errors of a statistically efficient estimator is provided

by the Cramer-Rao lower bound. This standard was utilized by the model parameter covariance matrix  $P_\theta(\hat{\theta}_N) = E[(\theta_0 - \hat{\theta}_N)(\theta_0 - \hat{\theta}_N)^T]$  of the difference between the true parameters  $\theta_0$  and estimated parameters  $\hat{\theta}_N$  as  $N$  tends to infinity. In general, the change of parameterization from a set of model parameters, given in an  $m \times 1$  dimensional vector  $\theta$ , to another set of physical parameters, given in a  $r \times 1$  dimensional vector  $\kappa$ , can be performed by a known  $r$ -dimensional functional relation

$$\kappa = f(\theta) \quad (25)$$

Since the number of physical parameters is less than the number of model parameters, obviously the accuracy and thus the sensitivity of  $\kappa$  is more significant than that of  $\theta$ . In addition, the functional relation (25) will in general be non-linear as in the case of the modal decomposition. Thus, to obtain a practically applicable approach, (25) is usually linearized using a first-order generalized Taylor expansion at the operating point  $(\hat{\kappa}_N, \hat{\theta}_N)$ , Andersen [5]

$$\begin{aligned} \kappa &= \hat{\kappa}_N + \left( \frac{\partial f(\theta)}{\partial \theta} \right)_{|\theta=\hat{\theta}_N} (\theta - \hat{\theta}_N) \\ &= \hat{\kappa}_N + J(\hat{\theta}_N)(\theta - \hat{\theta}_N) \end{aligned} \quad (26)$$

where  $J(\hat{\theta}_N)$  is a Jacobian matrix of partial derivatives which should be evaluated at the operating point  $\hat{\theta}_N$ . The covariance matrix  $P_\kappa(\hat{\kappa}_N)$  of the deviation of  $\hat{\kappa}_N$  from the true parameters can be estimated by

$$\begin{aligned} \hat{P}_\kappa(\hat{\kappa}_N) &= E[(\kappa_0 - \hat{\kappa}_N)(\kappa_0 - \hat{\kappa}_N)^T] \\ &= J(\hat{\theta}_N)P_\theta(\hat{\theta}_N)J^T(\hat{\theta}_N) \end{aligned} \quad (27)$$

The estimated covariance matrix  $\hat{P}_\theta(\hat{\theta}_N)$  obtained from (18) can then be inserted instead of  $P_\theta(\hat{\theta}_N)$ . This expression will only be accurate if  $\hat{P}_\theta(\hat{\theta}_N)$  is a good estimate of  $P_\theta(\hat{\theta}_N)$  and if the error due to the linear approximation is small, Andersen [5]. What remains is to calculate the Jacobian matrix  $J(\hat{\theta}_N)$ . This is in general impossible to do analytically even for small model structures, when the physical parameters of interest are the modal parameters. It is therefore necessary to rely on numerical differentiation. As an example, the elements of  $\hat{\kappa}_N$  can be defined as the estimated natural eigenfrequencies and associated damping ratios of the model.

$$\hat{\kappa}_N = [f_1 \quad \zeta_1 \quad f_2 \quad \zeta_2 \quad \dots \quad f_r \quad \zeta_{r+1}]^T \quad (28)$$

The functional relationship between these parameters and the model parameters is given by the eigenvalue problem (21) followed by the calculation of the modal parameters given in (24). This means that the resulting functional relation between the model and modal parameters is highly non-linear, and numerical differentiation must be applied. For further information on the practical considerations and the estimation of the uncertainties of the mode shapes, see Andersen [5].

## 5. EXAMPLE OF A STATISTICAL BASED DAMAGE DETECTION

This example illustrates the applications of system identification using ARMAV models in damage detection. This applications will be illustrated on a lattice steel test mast. An elevation of the 20 m high steel lattice test mast is shown in figure 1a. The four chord K-frame test mast with a  $0.9 \times 0.9$  m cross-section is bolted with twelve bolts, three for each chord, to a concrete foundation block. This block is founded on chalk and covered by sand. The mast is constructed with welded joints. At the top of the mast there are two plywood plates in order to increase the wind forces on the structure. In one of the lower diagonals, which is indicated in figure 1a, a damage has been simulated by introducing a crack and increasing its depth. The depth of the crack has been increased 4 times, see figure 1b. Before the damage is introduced the state of the structure is referred to as the virgin state. After the introduction of the damage the four different states of the structure are referred to as damage states. The mast has been equipped with 6 accelerometers shown in figure 1a. Three of them are mounted at the top of the mast and three approximately in the middle of it. Unfortunately, one of the accelerometers placed in the middle was damaged due to heavy rain, which means that the analysis has been performed with the remaining five accelerometers. Figure 1c shows a sketch of the location of the accelerometers and their sensitive directions are indicated with arrows. The damaged accelerometer has the number 2.2.

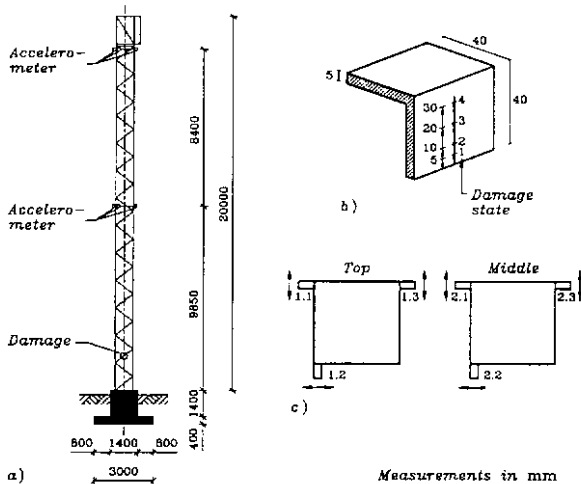


Figure 1: a) Elevation of the mast, where the location of the introduced damage and the six accelerometers is marked. b) The simulated damage is made by increasing the crack depth in four steps. These steps are referred to as damage states 1 to 4. c) The top and middle cross-section where the accelerometers are mounted. The sensitive directions of the accelerometers are indicated with arrows.

The data have been sampled with a sampling interval  $T = 0.02$  sec. Each measurement session lasted 160 sec, which implies a record length of  $N = 8000$  points. Initially, it is impossible to determine which of the modal parameters will be sensitive to damage. However, it is assumed that some of the estimated natural eigenfrequencies or the associated damping ratios are applicable as damage indicators. Since the mode

shapes are only described at a few points of the structure these will not be used as damage indicators.

State of the Structure	Cross-Sectional Reduction [%]	Crack Depth [mm]
Virgin state	0	0
1. Damage State	7	5
2. Damage State	13	10
3. Damage State	27	20
4. Damage State	40	30

Table 1: Definition of virgin and damage states.

As seen in figure 2 there are two closely spaced modes around 2 Hz and two closely spaced modes around 11.5 Hz.

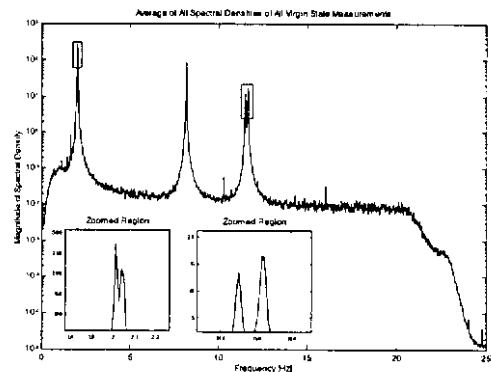


Figure 2: FFT-based spectral density of the response using all virgin state measurements. The auto-spectral densities and magnitudes of the cross-spectral densities of all measurement records have been averaged. All records have been normalized to unit standard deviation before averaging. The frequency resolution is 2048 between 0 and 25 Hz. The zoomed regions are the areas around the peaks at 2 Hz and 11.5 Hz.

The two modes located around 2 Hz are the first modes of bending in the two perpendicular directions, see figure 1. The mode located around 8 Hz is the first torsional mode of the structure, and the two modes located around 11.5 Hz are the second modes of bending. These observations are in accordance with prior research reported in Kirkegaard et al. [7], where the dynamic behaviour of the structure has been investigated. Because of the closely spaced modes the most sensible choice of model structure is an ARMAV model with as many channels as possible, i.e. 5 channels. In this way information from all sensors is used. Assuming that all 5 modes are underdamped and that noise is present in the measurements, it is necessary to use at least the ARMAV(2,2) model. However, the optimal order  $n$  has been selected on the basis of the Akaike FPE criterion as  $n = 3$ . This model structure has been used in all identifications. The actual system identifications have been performed using the non-linear PEM algorithm. The average values and sampled standard deviations of the modal parameter estimates of the five structural modes have

been calculated. From these values, it is also possible to estimate the coefficient of variation. The results are seen in tables 2 and 3.

Mode #	$f_i$ [Hz]	$\sigma_{f_i}$ [Hz] $\times 10^{-3}$	$v_{f_i}$ [Hz] $\times 10^{-3}$
1 (Bending)	2.014	2.49	1.24
2 (Bending)	2.044	2.63	1.28
3 (Torsion)	8.166	3.91	0.48
4 (Bending)	11.504	8.66	0.75
5 (Bending)	11.642	5.44	0.46

Table 2: Mean values  $f_i$ , sampled standard deviations  $\sigma_{f_i}$  and coefficients of variation  $v_{f_i}$  of the natural eigenfrequencies of the five structural modes.

Mode #	$\zeta_i$ [%]	$\sigma_{\zeta_i}$ [%]	$v_{\zeta_i}$
1 (Bending)	0.31	0.14	0.46
2 (Bending)	0.31	0.15	0.48
3 (Torsion)	0.10	0.05	0.48
4 (Bending)	0.18	0.20	1.07
5 (Bending)	0.13	0.06	0.45

Table 3: Mean values  $\zeta_i$ , sampled standard deviations  $\sigma_{\zeta_i}$  and coefficients of variation  $v_{\zeta_i}$  of the natural eigenfrequencies of the five structural modes.

As seen the coefficients of variation of the estimated natural eigenfrequencies are very small compared to the coefficients of variation of the estimated damping ratios. Since the coefficients of variation of the estimated damping ratios have values between 0.45 and 1.07 it implies that the estimates are very poor. It is therefore not worthwhile to use these estimates for damage detection.

Having completed the virgin state analysis, the analysis of the natural eigenfrequency estimates of the damaged states can begin. It is assumed that the structure shows linear and time-invariant behaviour in each damage state. This is a reasonable assumption since the size of the crack is extended between the last and the first measurement session of two consecutive damage states. If the size of the crack was increased significantly during a measurement session the system would perhaps show time-variant behaviour. From table 2, it is seen that the estimation inaccuracies for the natural eigenfrequencies result in coefficients of variation around  $1.0 \times 10^{-3}$ . Therefore, in the following analysis, natural eigenfrequency estimates with estimated coefficients of variation larger than  $1.0 \times 10^{-3}$  will be rejected from the analysis. In this way the uncertainties of the natural eigenfrequency estimates of the damaged states will qualitatively correspond to the estimates of the virgin state. At the present state, it has been established that the natural eigenfrequencies of the five structural modes can be estimated with a high degree of accuracy.

However, this does not mean that they can be used as damage indicators. They might simply not be sensitive to the damage. The easiest way to determine whether they are suitable as damage indicators or not is by plotting the estimated natural eigenfrequencies that have passed the rejection criteria described above. It turns out that only the third and fifth mode changes significantly. These changes are plotted in figures 3 and 4 together with the 95% confidence intervals based the estimated standard deviation obtained from the estimation of the ARMAV model.

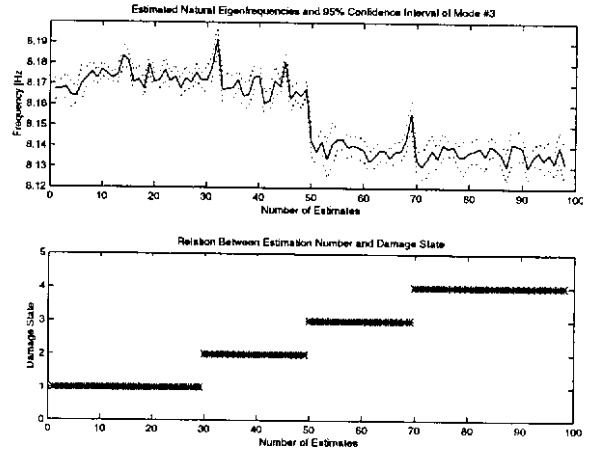


Figure 3: Estimated natural eigenfrequencies of the third mode that have passed the rejection criteria. The estimates are plotted together with their estimated 95% confidence interval. The relation between the estimates and the damage states is plotted below.

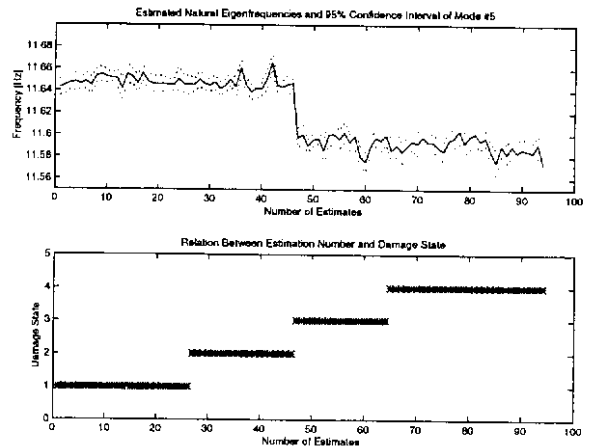


Figure 4: Estimated natural eigenfrequencies of the fifth mode that have passed the rejection criteria. The estimates are plotted together with their estimated 95% confidence interval. The relation between the estimates and the damage states is plotted below.

The observations made in figures 3 and 4 imply that the the third and fifth natural eigenfrequencies can be used as damage indicators for this particular damage. The question is at what damage state it is possible to detect the damage with a significant confidence.

In the following, it will be assumed that a damage has been detected if the confidence intervals of all the natural eigenfrequency estimates of a mode at some damage state are non-overlapping with the 95% confidence interval of the natural eigenfrequency of the same mode in the virgin state.

This detection approach can be utilized by plotting the estimated selected natural eigenfrequencies and their estimated 95% confidence intervals of the damage states together with the averaged natural eigenfrequency estimates and the estimated confidence intervals of the virgin state. In figure 5, this is done for the third natural eigenfrequency estimates, and in figure 6 for the fifth natural eigenfrequency estimates.

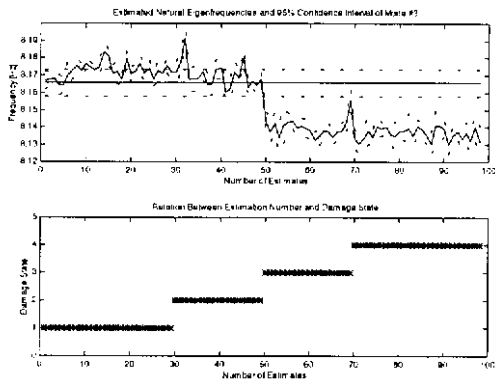


Figure 5: Estimated natural eigenfrequencies of the third mode and their estimated 95% confidence intervals.

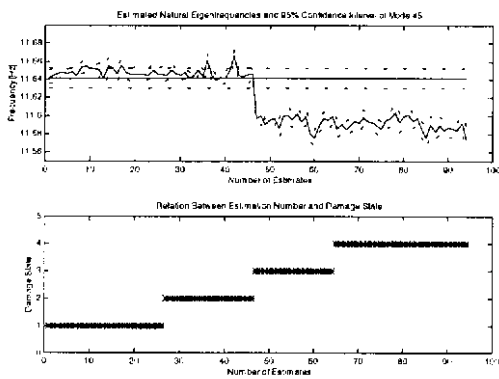


Figure 6: Estimated natural eigenfrequencies of the fifth mode and their estimated 95% confidence intervals.

In figure 5, it is seen that the confidence intervals become non-overlapping at the moment where damage state four is entered. However, already at damage state three the confidence intervals of the natural eigenfrequency estimates of the fifth mode are completely non-overlapping with the virgin state confidence interval, see figure 6. So due to the above definition of a significant damage, it can be concluded that the actual damage has been detected when it entered the third damage state. The detected changes of the natural eigenfrequencies are so significant that they are probably caused by a structural change. However, the modal parameters can also exhibit small changes due to fluctuations in the ambient environment. When e.g. the ambient temperature changes, thermal expansion effects and changes of the stiffness will occur. The effects of fluctuating ambient temperatures on this particular mast have been investigated in Kirkegaard et al. [7], and a technique based on Kalman filtering for removal of such an influence has been proposed in Andersen et al. [4].

## 6. CONCLUSIONS

In this paper a simple statistical approach for damage detection of civil engineering structures has been given. This approach is based on estimation of an ARMAV model using the prediction error method. By means of this method it is possible to obtain accurate estimates of the modal parameters and estimates of their associated standard deviations. By using these estimated standard deviations, it is actually possible to detect a damage in a statistical sense. This has been illustrated on a lattice steel mast that exhibits a growing crack in one of its lower diagonals. When this crack has reached a depth of 20 mm the change of the third and the fifth natural eigen-frequencies of the structures becomes significant within a 95% confidence.

## 7. ACKNOWLEDGEMENT

The financial support from the Danish Technical Research Council is gratefully acknowledged.

## 8. REFERENCES

- [1] Andersen, P., R. Brincker & P.H. Kirkegaard: *On the Uncertainty of Identification of Civil Engineering Structures using ARMA Models*. Proc. IMAC 13, Nashville, Tennessee, 1995.
- [2] Andersen, P., R. Brincker & P.H. Kirkegaard: *Theory of Covariance Equivalent ARMAV Models of Civil Engineering Structures*. Proc. IMAC 14, Dearborn, Michigan, USA, 1996.
- [3] Andersen, P., P.H. Kirkegaard & R. Brincker: *System Identification of Civil Engineering Structures using State Space and ARMAV Models*. Proc. ISMA 21, Leuven, Belgium, 1996.
- [4] Andersen, P., P.H. Kirkegaard & R. Brincker: *Filtering Out Environmental Effects in Damage Detection of Civil Engineering Structures*. Proc. IMAC 15, Orlando, Florida, USA, 1997.
- [5] Andersen, P.: *Identification of Civil Engineering Structures using Vector ARMA Models*. Ph.D.-Thesis, Aalborg University, Denmark, ISSN 1395-7953 R9724, 1997.
- [6] Doebling, S.W., C.R. Farrar & R.S. Goodman: *Effects of Measurement Statistics on the Detection of Damage in the Alamosa Canyon Bridge*. Proc. IMAC 15, Orlando, Florida, USA, 1997.
- [7] Kirkegaard, P.H. & A. Rytter: *An Experimental Study of a Steel Lattice Mast under Natural Excitation*. Proc. IUTAM Symposium on Identification of Mechanical Systems, Wuppertal, Germany, 1993.
- [8] Kirkegaard, P.H. & P. Andersen: *Use of Statistical Information for Damage Assessment of Civil Engineering Structures*. Proc. IMAC 16, Santa Barbara, California, USA, 1998.
- [9] Ljung, L.: *System Identification - Theory for the User*. Prentice-Hall, Englewood Cliffs, New Jersey, 1987.
- [10] Rytter, A.: *Vibrational Based Inspection of Civil Engineering structures*. Ph.D.-Thesis, Aalborg University, Denmark, ISSN 0902-7513 R9314, 1993.