

AMBIENT MODAL TESTING OF THE VESTVEJ BRIDGE USING RANDOM DECREMENT

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Abstract *This paper presents an ambient vibration study of the Vestvej bridge. The bridge is a typical Danish two-span concrete bridge which crosses a highway. The purpose of the study is to perform a pre-investigation of the dynamic behaviour to obtain information for the design of a demonstration project concerning application of vibration based inspection of bridges. The data analysis process of ambient vibration testing of bridges has traditionally been based on auto and cross spectral densities estimated using an FFT algorithm. In the pre-analysis state the spectral densities are all averaged to obtain the averaged spectral densities (ASD). From the ASD the eigenfrequencies of the structure can be identified. This information can be used in the main analysis, where all modal parameters are extracted from the spectral densities. Due to long cabling and low response levels (small ambient loads) the response measurements might have a low signal to noise ratio. Thus, it might be difficult clearly to identify physical modes from the spectral densities. The Random Decrement (RD) technique is another method to perform the data analysis process in the time domain only. It is basically a very simple and easily implemented technique. In this paper it is demonstrated how the RD technique can be used in the pre-analysis state in combination with the FFT algorithm, and how the technique can be used in a full analysis.*

Nomenclature

\mathbf{C}	Symmetric positive definite damping matrix.
$D_{YX}(\tau)$	Random Decrement function.
$\mathbf{F}(t)$	Excitation vector.
$\mathbf{h}(t)$	Impulse response matrix/function.
$\mathbf{H}(\omega)$	Frequency response matrix/function.
\mathbf{K}	Symmetric positive definite stiffness matrix.
\mathbf{m}	Modal mass matrix.
\mathbf{M}	Diagonal positive definite mass matrix.
N	Number of triggering points.
$p_X(x)$	Density function of X .
$R_{YX}(\tau)$	Cross correlation function

t, τ	Time variables.
$T_{X(t)}^P$	Positive point triggering.
$\mathbf{W}(t)$	White noise excitation vector.
$X(t), Y(t)$	Stochastic processes.
$x(t), y(t)$	Realizations of $X(t), Y(t)$, measurements.
ω	Frequency.
Λ	Diagonal matrix with eigenvalues.
Φ	Mode shape matrix.
FRM	Frequency Response Matrix.
IRM	Impulse Response Matrix
MCF	Modal Confidence Factor.
PTD	Polyreference Time Domain.
RD	Random Decrement.

1 Introduction

The ambient vibration study of the Vestvej bridge forms the initial part of a demonstration project concerning the application of vibration based inspection to bridges. The aim of the project is to perform a continuous on-the-line surveyance of the bridge using the RD technique. The RD technique is capable of handling large data quantities, since it transforms the response into short data segments, correlation functions. In order to obtain information about the modal parameters of the bridge, ambient vibration tests are carried out. The tests are described in this section. The ambient vibrations have been collected on March 25 1997 and June 4 1997. In this section the analysis of the data collected on June 4 1997 are described in detail, whereas only some of the results of the data analysis of the data collected the 25/03 1997 are presented. The analysis is also reported in Asmussen et al. [2], [3], [4] and summarized in [1].

The Vestvej bridge is shown in fig. 1.

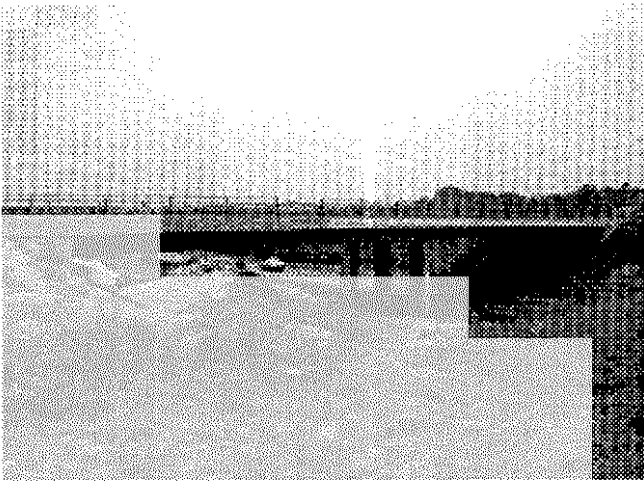


Figure 1: *The Vestvej bridge.*

A description of the experimental test of the bridge is given in section 2 . The theory behind and the implementation of the RD technique is described in section 3 . Section 4 explains the data analysis methodology and the results are presented in section 5 . The paper is finished with a conclusion.

2 Ambient Tests of the Vestvej Bridge

The Vestvej bridge is crossing the highway from Aalborg to Frederikshavn between Vodskov and Langholt just north of Aalborg, Denmark. The main geometry of the bridge is shown in fig. 14. The western 2/3 of the bridge was erected in 1986 and the remaining 1/3 in 1996. The bridge deck is made of post-tensioned concrete.

The bridge is mainly loaded by the vehicles on the bridge. But also the vehicles passing under the bridge on the highway can contribute to the vibrations of the bridge. Furthermore, wind can generate vibrations of the bridge. Since all these forces are unmeasurable and ambient the vibration study is denoted ambient.

The measurement system is an 8 channel system, which means that the bridge response can be recorded, A/D-converted and saved to disk from 8 channels or measurement locations simultaneously. In order to measure the vibrations of the bridge at more than 8 locations and still preserve the possibility of estimating mode shapes, several setups with two reference locations, for the purpose of safety and flexibility, are collected. The chosen measurement locations and the reference locations are seen in fig. 2. The accelerometers are Schaewitz type at

20V/g, full range of ± 0.25 g and secured in watertight steel boxes.

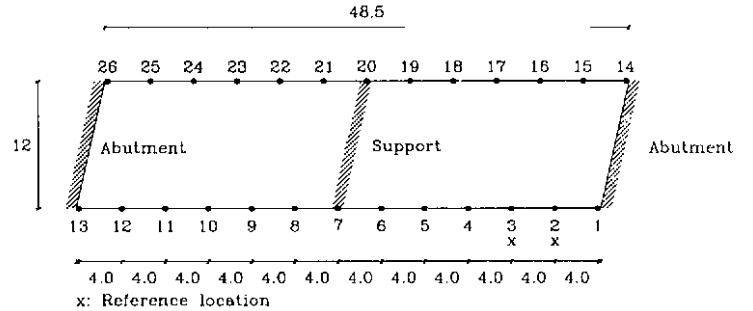


Figure 2: *Outline draft of the Vestvej bridge and the measurement locations. Distances in m.*

Since 26 measurement locations are chosen 5 different setups with data are collected.

The measurements were sampled at 160 Hz for 900 seconds. The data were detrended and decimated twice (this includes lowpass digital filtering) before saved to disk. The resulting sampling frequency is thereby reduced to 80 Hz and the number of points in each measurement is 72000. Detrending removes any linear trend from the data and decimation reduces the sampling frequency and suppresses the noise in the records.

3 The Random Decrement Technique

It is assumed that the vibrations of the bridge can be modelled by an equivalent MDOF linear viscously damped system.

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t) \quad (1)$$

The system is assumed to be time-invariant during the measurement period and the forces are assumed to be stationary. Usually it is assumed that the force vector is a white noise stochastic vector process. In order to generalize the modelling of the forces, it will be assumed that the force vector $\mathbf{F}(t)$ can be modelled by a white noise stochastic vector process passed through a pseudo force filter or a shaping filter, see Ibrahim et al. [5].

$$\mathbf{F}(\omega) = \mathbf{H}_F(\omega)\mathbf{W}(\omega) \quad (2)$$

$$\mathbf{F}(t) = \int_{-\infty}^t \mathbf{h}_F(t - \tau)\mathbf{W}(\tau)d\tau \quad (3)$$

where \mathbf{H}_F and \mathbf{h}_F are the FRM and IRM of the pseudo force filter. Subscript F refers to the modelling of the force. The IRM of the filter is assumed to be given by

$$\mathbf{h}_F(t) = \mathbf{\Phi}_F e^{\Lambda_F t} \mathbf{m}_F^{-1} \mathbf{\Phi}_F^T \quad (4)$$

To illustrate the effect of the pseudo force filter a filter with an SDOF is considered. Figure 3 shows the spectral density and auto correlation function of the white noise process, $W(t)$, and the spectral density and auto correlation function of the resulting force passed through the SDOF filter.

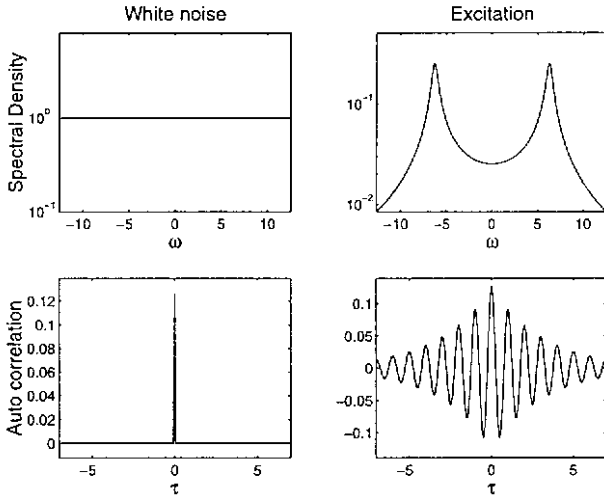


Figure 3: The effect of modelling the excitation as a filtered white noise process, where the filter has a SDOF.

Using this modelling of the excitation it follows that the structural response is Gaussian distributed. The system, which is identified from the ambient response, is a combination of the modes of the structural system and the pseudo force filter. These remain uniquely identifiable, see Ibrahim et al. [5].

The Random Decrement technique is used for the data analysis of the measurements. The auto and cross RD functions are defined as conditional mean values.

$$D_{XX}(\tau) = E[X(t+\tau)|T_{X(t)}] \quad (5)$$

$$D_{YX}(\tau) = E[Y(t+\tau)|T_{X(t)}] \quad (6)$$

where $T_{X(t)}$ is the triggering condition. The first index of D refers to the process where the averaging is performed and the second index refers to the process where the triggering condition is fulfilled. The time variable τ can be both positive and negative.

A description of the different triggering conditions possible can be seen in Asmussen [1]. In this analysis the positive point triggering condition is used

$$T_{X(t)}^P = \{a_1 \leq X(t) < a_2\} \quad (7)$$

where a_1, a_2 are the triggering levels. The triggering levels should fulfil $a_2 > a_1 > 0$ or $a_1 < a_2 < 0$. It is recommended to use $a_1 > \sigma_X$ or $a_1 < -\sigma_X$, see Asmussen [1].

If the processes $X(t)$ and $Y(t)$ are stationary and jointly Gaussian distributed with zero mean it can be shown that the RD functions are proportional to the correlation functions of the processes

$$D_{XX}(\tau) = \frac{R_{XX}(\tau)}{\sigma_X^2} \cdot \tilde{a} \quad (8)$$

$$D_{YX}(\tau) = \frac{R_{YX}(\tau)}{\sigma_X^2} \cdot \tilde{a} \quad (9)$$

$$\tilde{a} = \frac{\int_{a_1}^{a_2} x p_X(x) dx}{\int_{a_1}^{a_2} p_X(x) dx} \quad (10)$$

By scaling the RD functions with the resulting triggering level \tilde{a} the RD functions become proportional to the correlation coefficient functions of $X(t)$ and $Y(t)$. Vandiver et al. [6] firstly linked RD functions with correlation functions. Their results were extended by Brincker et al. [7].

The RD functions are unbiased estimated as

$$\hat{D}_{XX}(\tau) = \frac{1}{N} \sum_{i=1}^N x(t_i + \tau) | a_1 \leq x(t_i) < a_2 \quad (11)$$

$$\hat{D}_{YX}(\tau) = \frac{1}{N} \sum_{i=1}^N y(t_i + \tau) | a_1 \leq x(t_i) < a_2 \quad (12)$$

where N is the number of triggering points and $x(t), y(t)$ are realizations of $X(t)$ and $Y(t)$.

The analogy between the RD functions and the spectral densities estimated using an FFT algorithm is obviously due to the Wiener-Khinchine relations. In the analysis of several simultaneously recorded measurements, say n , it is possible to estimate n^2 RD functions corresponding to estimating the full correlation matrix of the measurements.

If the modelling of the structural system and the excitation in eqs. (1) - (4) is valid, a column of the correlation matrix of the response process, $X(t)$, can be described by

$$\mathbf{R}_{XX}^i(\tau) = \Phi e^{\Lambda \tau} \mathbf{m}^{-1} \mathbf{q} \quad (13)$$

where \mathbf{R}_{XX}^i denotes the i th column of the correlation matrix at the time τ , Φ contains the mode shapes of the structure and scaled modes of the filter, Λ contains the eigenvalues of the structure and the filter, \mathbf{m} is a normalizing matrix equivalent to the modal mass matrix, and \mathbf{q} is a scaling matrix dependent on Φ and the covariance

structure of $\mathbf{W}(t)$, see Asmussen [1]. A corresponding result can be seen in James III et al. [8]

Equation 13 corresponds to a scaled version of a free decay and/or the IRM. This means that on the given assumptions, the modal parameters can be extracted from the RD functions using methods based on free decays, such as the Polyreference Time Domain technique (PTD) see Vold et al. [9]. The PTD technique is used to extract the modal parameters from the RD functions in this analysis.

4 Data Analysis Methodology

The RD technique is applied for the analysis of the acceleration measurements. The data have a high noise content as seen in fig. 4. It seems as if a noise band between approximate -0.2 g and 0.2 g is added to the structural response.

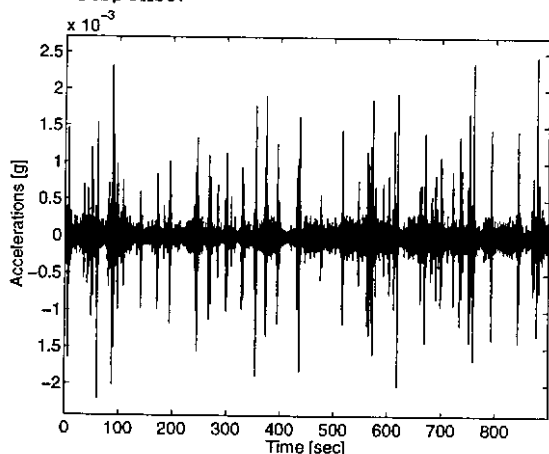


Figure 4: Typical acceleration record from the Vestvej bridge.

The reason is that only a few vehicles are crossing the bridge during the 900 second record period. This means that the accelerations are mainly due to the wind and the vehicles passing under the bridge. Furthermore, long cabling is used (20m-100m) and the sensitivity of the accelerometers is high compared to the small accelerations of the concrete bridge, see fig. 4. This makes the analysis of the data challenging.

Due to the high content of noise it is chosen to use the positive point triggering condition, so that sufficient triggering points are available. On the other hand, using triggering bounds close to zero might introduce false triggering points, which will increase the uncertainty of the RD functions. The triggering levels should fulfil these three conditions:

- No false triggering points should be introduced.

- The number of triggering points should be sufficient to average out the contribution from the noise.
- The number of triggering points should be restricted so that the estimation time is reasonable.

A typical response measurement is considered. Two different sets of triggering bounds are investigated: $[a_1 \ a_2] = [0 \ 1.5\sigma_X]$ and $[a_1 \ a_2] = [1.5\sigma_X \ \infty]$. The auto RD functions are estimated for 200 positive and negative time lags. The number of triggering points was 33200 and 2780 and the corresponding estimation times in CPU were 2.69 and 0.44. As expected, $[a_1 \ a_2] = [1.5\sigma_X \ \infty]$ has the lowest number of triggering points and is thereby the fastest. In order to evaluate the different estimates the symmetry relation for correlation functions of stationary processes

$$R_{XY}(\tau) = R_{YX}(-\tau) \quad (14)$$

is used to calculate the error and the average of the autocorrelation function for positive time lags only, see Asmussen [1]. The results are shown in figs. 5 and 6.

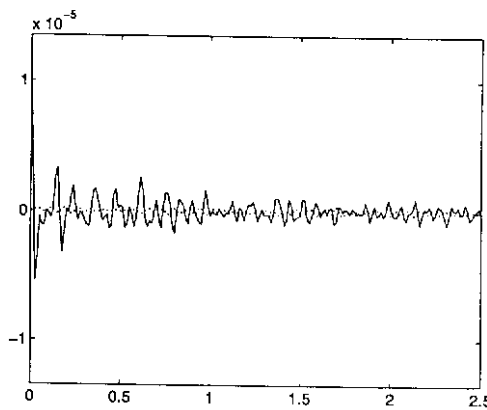


Figure 5: The average (full line) and error (dotted line) estimate of an RD function using $[a_1 \ a_2] = [0 \ 1.5\sigma_X]$.

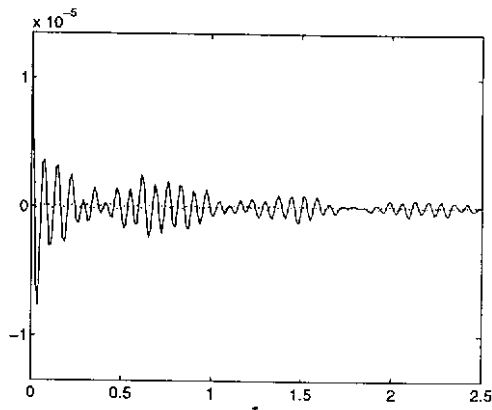


Figure 6: The average (full line) and error (dotted line) estimate of an RD function using $[a_1 \ a_2] = [1.5\sigma_X \ \infty]$.

The difference is significant and the error function shows that although there are only 0.15 times as many triggering points using $[a_1 \ a_2] = [1.5\sigma_X \ \infty]$ the estimate is far more accurate. This is positive, since the most accurate approach is thereby also the fastest approach. So, the triggering levels are chosen as $[a_1 \ a_2] = [1.5\sigma_X \ \infty]$.

For each of the five setups the full correlation matrix is estimated using the positive point triggering condition. After the estimation of the RD functions and before the modal parameters are extracted it is common in ambient testing of bridges to perform some kind of pre-analysis to have an idea about the number of structural modes present in the measurements. Such an analysis is developed for ambient testing based on FFT estimated spectral densities. The method is denoted Average Spectral Densities (ASD) and the idea is simply to average the spectral densities of all measurements. The ASD will strongly indicate the number of modes and the corresponding frequencies. Figure 7 shows the ASD calculated using FFT of the measurements.

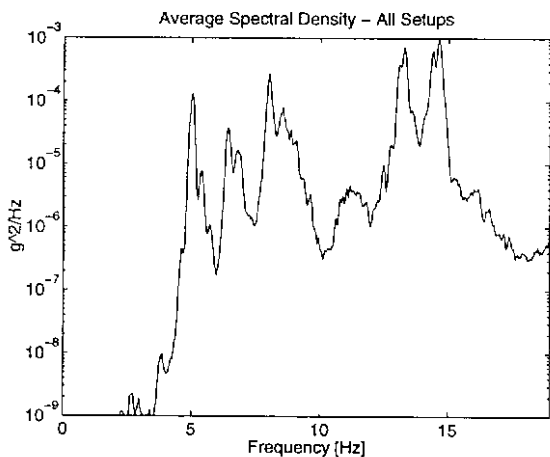


Figure 7: The ASD calculated from the spectral densities of the measurements.

The idea behind this approach is adapted to the RD technique. Instead of averaging the spectral densities of the measurements, the Fourier transform of the estimated RD functions is calculated and averaged. The estimate of the RD based ASD is shown in fig. 8.

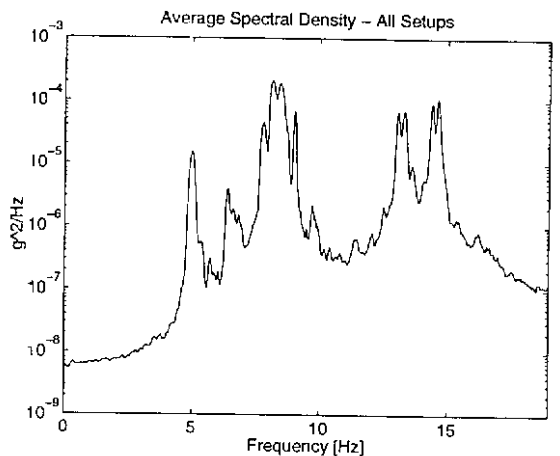


Figure 8: The ASD calculated from the FFT of the RD functions of the measurements.

From fig. 8 the structural modes with the following frequencies are detected.

5.00	6.40	6.62	6.81	7.80	8.13
8.44	9.02	13.06	13.37	14.39	14.61

Table 1: Natural frequencies in Hz of the Vestvej Bridge.

It seems as if the ASD based on the RD functions provides a better basis for detecting structural modes. The reason is that the ASD from RD functions is based on averaging in both time and frequency domain, whereas the ASD based on the spectral densities of the measurements is only based on averaging in the frequency domain. This is an important relation for the RD technique.

The frequencies shown in table 1 are the result of a pre-analysis where the information from all measurements is used. In the full analysis it is not certain that all modes can be identified, since some of the modes may have a low contribution at several measurement points. Thereby a requirement of high confidence for the modes are violated and the mode can only be identified at some of the setups

5 Results

The modal parameters are extracted using PTD from the estimated correlation matrices belonging to the different measurement setups. The aim is to estimate the mode shapes corresponding to the frequencies in table 1, but the noise content in the data is high so not all mode shapes can be estimated at a high confidence level from all 5 setups. The influence of the number of points used from the RD functions and the model order is investigated by changing the number of modes from 25 to 30 and varying the number of points in the RD functions

from 100 to 120. The following restriction has been applied in order to detect a structural mode from the noise and force modes: $\zeta < 0.05$, $|MCF| > 0.9$ $\angle MCF < 10^\circ$, where MCF is the Modal Confidence Factor, see Ibrahim [10], Vold [11]

Parameter	Date	1	2	3	4	5
f [Hz]	25/03	5.16	6.64	8.31	14.01	15.38
f [Hz]	04/06	5.02	6.58	8.04	13.23	14.56
ζ [%]	25/03	1.85	4.07	3.04	2.15	2.42
ζ [%]	04/06	1.21	3.52	2.45	1.20	1.17

Table 2: *Eigenfrequencies and damping ratios of the Vestvej Bridge.*

As seen not all mode shapes could be detected with sufficient confidence in all setups. The corresponding mode shapes are shown in figs. 9 - 13.

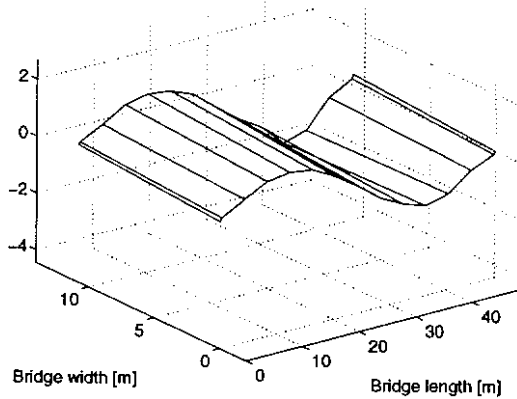


Figure 9: *First mode shape of the Vestvej bridge.*

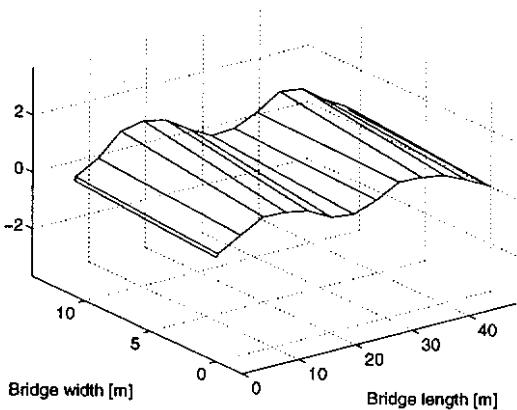


Figure 10: *Second mode shape of the Vestvej bridge.*

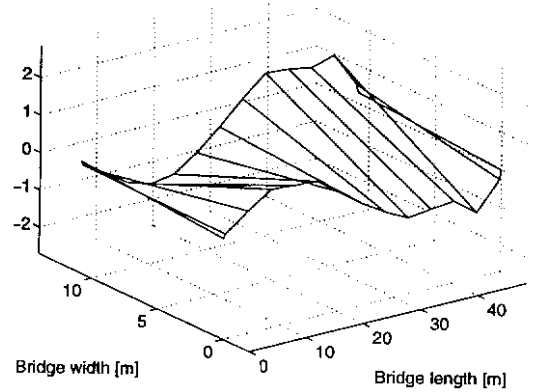


Figure 11: *Third mode shape of the Vestvej bridge.*

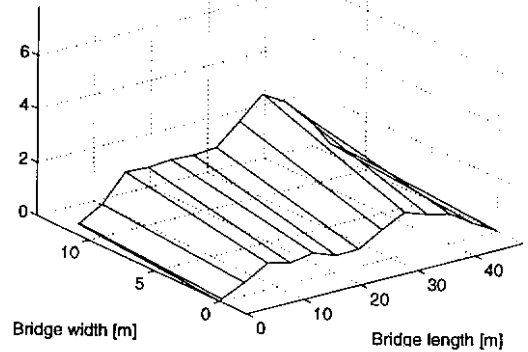


Figure 12: *Fourth mode shape of the Vestvej bridge.*

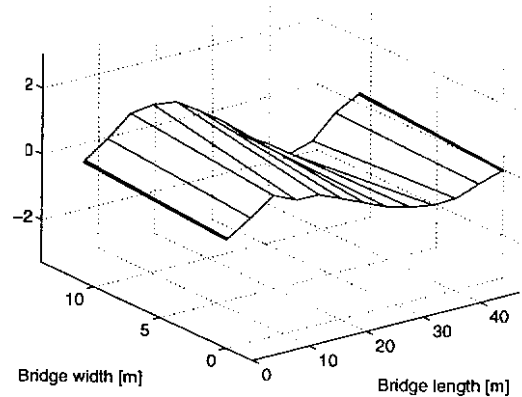


Figure 13: *Fifth mode shape of the Vestvej bridge.*

6 Conclusions

An ambient vibration study of the Vestvej bridge has been performed. It has been demonstrated how the data analysis process can be performed using the Random

Decrement technique. It was possible to estimate the eigenfrequencies of the bridge and some of the corresponding mode shapes with high confidence. The bridge can be used for a demonstration project concerning vibration based inspection, see Rytter [12]. It is recommended to do the surveyance continuously at several locations of the bridge using the RD technique. This makes it possible to obtain more accurate RD functions and at the same time use as much information from the vibrations as possible.

7 Acknowledgement

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Figure 14: The Vestvej bridge.

