

Modal testing of mechanical structures subject to operational excitation forces

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Abstract

Operational Modal Analysis also known as Output Only Modal Analysis has in the recent years been used for extracting modal parameters of civil engineering structures and is now becoming popular for mechanical structures. The advantage of the method is that no artificial excitation need to be applied to the structure or force signals to be measured. All the parameter estimation is based upon the response signals, thereby minimising the work of preparation for the test. This test case is a controlled lab set-up enabling different parameter estimation methods techniques to be used and compared to the Operational Modal Analysis.

For Operational Modal Analysis two different estimation techniques are used: a non-parametric technique based on Frequency Domain Decomposition (FDD), and a parametric technique working on the raw data in time domain, a data driven Stochastic Subspace Identification (SSI) algorithm. These are compared to other methods such as traditional Modal Analysis.

1. Introduction

Operational Modal Analysis often called *Ambient Modal* or *Output only Modal* is a technique for modal parameter estimation without knowing the input loading force. The method has been frequently used for parameter estimation on civil structures such as bridges and towers where artificial excitation and determination of forces exhibits a problem. In this paper the use of the Operational Modal Analysis method for mechanical structures will be discussed.

The test case is a lab set-up where the environment is well controlled so the Operational Modal Analysis can be compared to modal analysis results obtained from traditional input output methods.

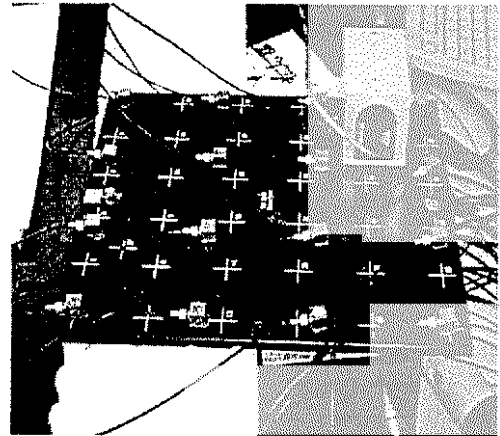


Figure 1 Test set-up A plate structure and electrical motor

Following tests were done:

- Modal Test using measured input from the exciter for estimation by conventional input/output modal estimation technique
- For the Operational Modal Analysis two tests were done:
 - Full run up/down of the motor 720-6000 RPM
 - Smaller RPM variation of the motor around 6000 RPM.

For the estimation of modal parameters by input/output modal analysis a handheld exciter on a corner point excites the plate, and the loading force applied to the structure is measured by means of a force transducer. 16 accelerometers were mounted on the plate and the frequency response between the input DOF and all the output DOFs are measured for use in the input output modal estimation.

During all the measurements a small motor is mounted on the plate (see Figure 1). This motor which is used as excitation source during the operational test has been introduced an amount of unbalance so the rotation will generate vibration force of 1st harmonic. Besides this also a family of higher harmonics are introduced.

2. Test Conditions

2.1 Instrumentation

For acquiring the data a Brüel & Kjaer PULSE multianalyser system was used together with Brüel & Kjaer modal accelerometers. For the artificial excitation a handheld exciter Brüel & Kjaer type 5961 together with force transducer type 8203 was used. The excitation equipment as well as the accelerometers were selected with the aim of keeping the mass loading down.

The PULSE system was set up with an FFT analyser running in parallel with a time capture analyser. The two analysers use the same synchronous time samples.

The FFT analyser acquires spectral information of the response during the operation of the motor. At the same time as raw time data are stored on the disk for later processing by the operational modal esti-

maters. The advantage of the multiprocessing is an on-line indication of the response spectra at the same time as time data are stored for the Operational Modal Analysis.

2.2 Data Processing

The frequency response functions were acquired using a handheld shaker with a miniature force transducer Brüel & Kjaer type 8203. This force transducer has an effective seismic mass of 2.1 gram (including pre-loading nuts). This low dynamic mass keeps the system invariant between the different tests.

Besides the real-time and online processing performed by the PULSE system two postprocessing packages were used:

2.2.1 Operational Modal Analysis developed by *Structural Vibrations Solutions*

For the Operational Modal Analysis method the software above were used for estimation of modal parameters.

The package uses two different techniques: a non-parametric technique based on Frequency Domain Decomposition (FDD), and a parametric technique working on the raw data in time domain, a data driven Stochastic Subspace Identification (SSI) algorithm.

The package draw use of different validation tools so the results derived from the different techniques can be compared.

2.2.2 ME'scope developed by *Vibrant Technology*

The FRF's measured during the Conventional modal test were analysed using the Rational Fraction Polynomial MDOF curvefitting provided by ME'scope.

3 Background

The **Rational Fraction Polynomial** curvefitting, which is one of the classical methods, is described in ref. [5]. This combined with animation of the mode-shapes, provides the modal model to be compared to the results from the operational modal analysis

Operational Modal Analysis has included a range of estimators divided in two main groups: Frequency

Domain Decomposition and Stochastic Subspace Identification.

The **Frequency Domain Decomposition** (FDD) technique used for the Operational Modal Analysis is an extension of the classical frequency domain approach. The classical approach is based on signal processing using the Discrete Fourier Transform, and is using the fact, that relatively lightly damped and well separated modes can be estimated directly from the power spectral density matrix at the peak. As the power spectral density matrix does not hold the full information especially in the case of close modes of the modal model, singular value decomposition is utilised in the method, ref. [1] and ref.[9].

Stochastic Subspace Identification (SSI) is a class of techniques that are all formulated and solved using state space formulations of the form

$$\begin{aligned}x_{t+1} &= Ax_t + w_t \\ y_t &= Cx_t + v_t\end{aligned}$$

where x_t is the Kalman sequences that in SSI is found by a so-called orthogonal projection technique, Overschee and De Moor [3]. Next step is to solve the regression problem for the transition matrix A , the observation matrix C , and for the residual sequences w_t and v_t . Finally, in order to complete a full covariance equivalent model in discrete time, the Kalman gain matrix K is estimated to yield

$$\begin{aligned}\hat{x}_{t+1} &= A\hat{x}_t + Ke_t \\ y_t &= C\hat{x}_t + e_t\end{aligned}$$

It can be shown, Brincker and Andersen [2], that by performing a modal decomposition of the A matrix as $A = V[\mu_i]V^{-1}$ and introducing a new state vector $z_t = V^{-1}\hat{x}_t$, the equation can also be written as

$$\begin{aligned}z_{t+1} &= [\mu_i]z_t + \Psi e_t \\ y_t &= \Phi z_t + e_t\end{aligned}$$

where $[\mu_i]$ is a diagonal matrix holding the discrete poles related to the continuous time poles λ_i by $\mu_i = \exp(\lambda_i \Delta t)$, and where the matrix Φ is holding the left hand mode shapes (physical, scaled mode shapes) and the matrix Ψ is holding the right hand mode shapes (non-physical mode shapes). The right

hand mode shapes are also referred to as the initial modal amplitudes, Juang [4].

The specific technique used in this investigation is the Principal Component algorithm, see Overschee and De Moor [3].

4 Results from Data acquisition

Three tests were performed:

- Motor run up/down between 720 and 6000 RPM (Operational Modal Analysis)
- Motor run varying between 5520 and 6060 RPM (Operational Modal Analysis)
- Frequency Response Functions (Shaker Excitation)

Figure 2 and Figure 3 shows contour plots of FFT spectra of one of the responses from the two operational tests. The plot type is similar to what is obtained in the popular waterfall plots and is very useful to identify the resonances and orders. The plots shows response autospectra as function of time with amplitude indicated by the colour coding.

Figure 2 shows the contour plot for a full run up/down between 720 RPM and 6000 RPM. The analysis frequency span of the FFT was set to 0-3.2 kHz. The motor was introduced some amount of unbalance causing a vibration the fundamental frequency together with higher harmonics giving excitation of the plate modes. From the plot it is seen that a range of resonances are excited, not only by the first harmonic but also by the higher harmonics while they pass through the resonances. As we use a broad RPM variation of the motor the family of harmonics will cover the whole frequency range. On Figure 2 the skewed lines indicates orders and the vertical (constant frequency) indicate resonances.

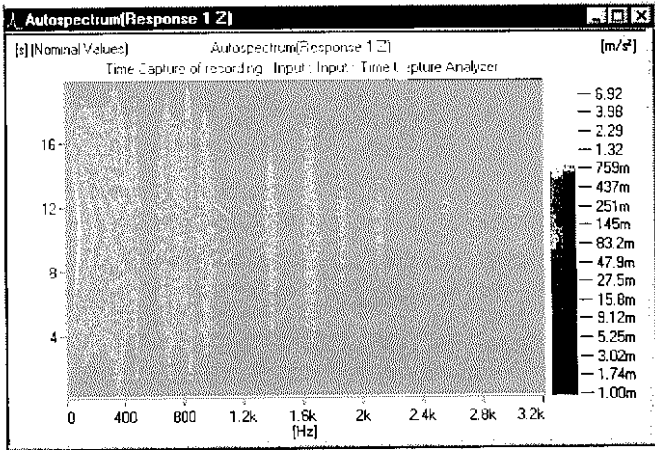


Figure 2 Contour plot from a broad run up/down

Figure 3 shows the contour of one of the response spectra during the test with motor speed varying between 5520 and 6060 RPM. In this case the frequency variation of the harmonics is small, which means that all individual resonances are not excited by a harmonic.

This plot was measured with the same analysis frequency span (3.2 kHz) but the plot is expanded in order to distinguish the modes more clearly.

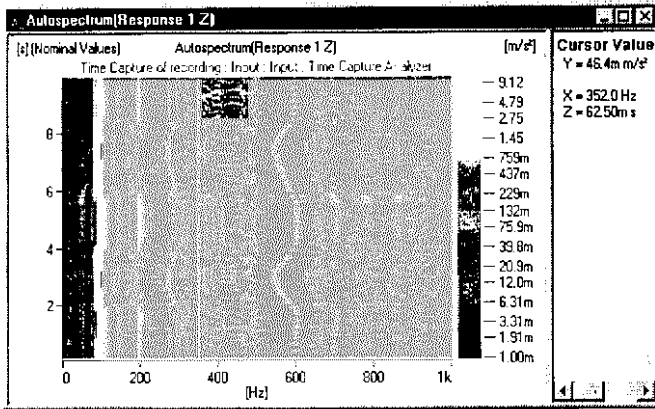


Figure 3 Contour plot from a narrow variation of the motor speed

Due to the narrow RPM variation, not all modes are excited equal. Especially the mode at 352 Hz (indicated by the cursor) is not excited or only excited at a very limited part of the test time.

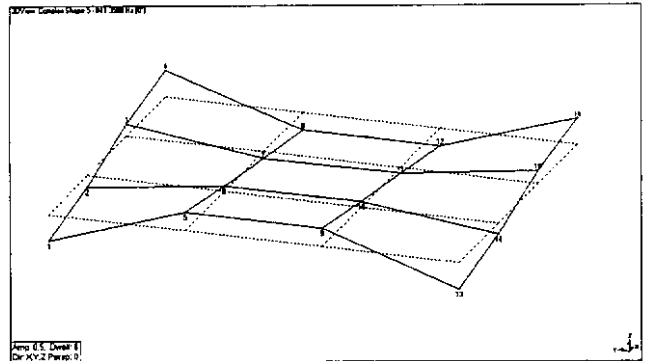
5. Input Output Modal Analysis

The direct Active-X supported data transfer between PULSE and ME'scope was used for making the FRFs available in ME'scope. The standard Rational Fraction Polynomial curvefitter was used fitting 2-3 modes at a time. A total of 13 modes were identified.

The estimation results for frequency and damping are seen in Table 1. Below is seen an animation of the 5th mode at 841 Hz which is one of the modes as expected from a plate structure.

Shape	Frequency	Damping (%)
1	351 Hz	1.38
2	476 Hz	0.55
3	698 Hz	0.76
4	825 Hz	0.72
5	945 Hz	0.73
6	1090 Hz	0.76
7	1619 Hz	0.71
8	1666 Hz	0.59
9	1863 Hz	0.80
10	2171 Hz	0.74
11	2553 Hz	0.78
12	2753 Hz	0.76
13	3062 Hz	1.03

Table 1 Results from Modal Analysis



6. Operational Modal Analysis

The data used for the Operational Modal Analysis were acquired in the PULSE time capture analyser as raw time data.

Before transferring into the Operational Modal Analysis program, it may be convenient to edit the time series in order to disregard improper measurement sequences in the analysis. This is in general done in PULSE before the transfer in order to minimise the data.

After having transferred the measurement to the Operational Modal Analysis program postprocessing tools are provided for further decimation and filtering. All the data in the following have been filtered with a 10th order 200 Hz HP filter. This was done in order to lower the level of the dominating lower harmonics, which only excited the rigid body modes not dealt with in this analysis.

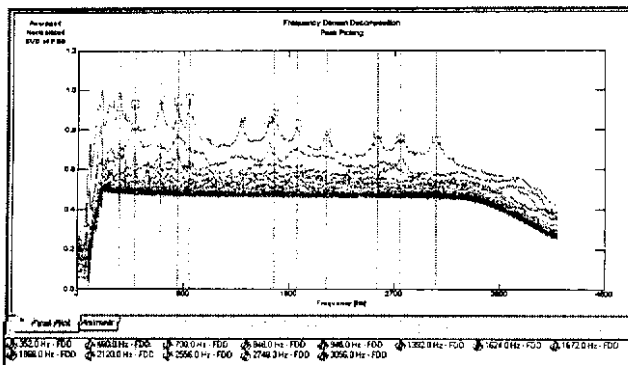


Figure 4 FDD of run up/down

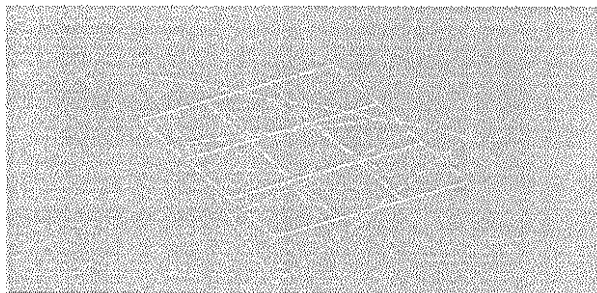


Figure 4 shows the identification of the modes by means of the FDD method, and associated example of an animation of the first flexible mode.

The result in Figure 4 was taken from the full range run up/down (720 RPM – 6000 RPM)

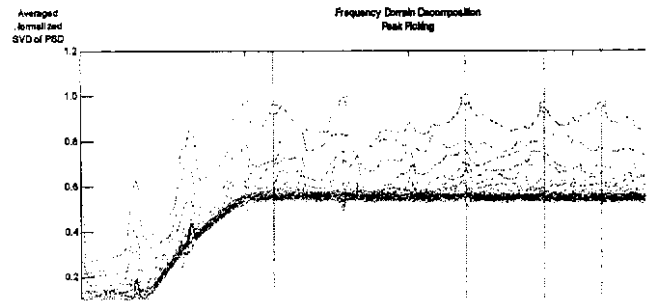


Figure 5 FDD of the narrow motor variation

Figure 5 shows the FDD estimator from the narrow motor variation (from 5520 RPM to 6060 RPM). It is seen that the first flexible mode is more difficult to identify, than in Figure 4. Note that the frequency axis in fig. 5 has been expanded.

The difficulty in identifying the first flexible mode is due to the close peak originating from the forced created vibration of the 4th harmonic.

Fig 6 show a stabilisation diagram calculated using one of the SSI methods. The unweighted principal component method for the full range run up/down.

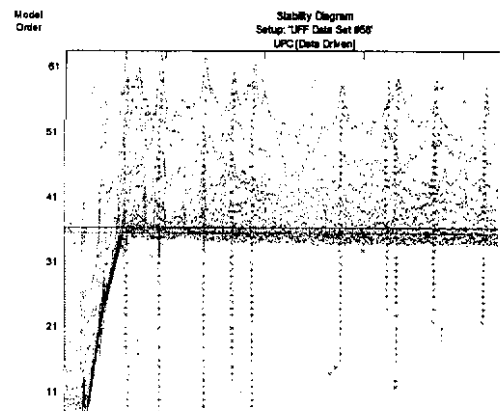


Figure 6 Stabilisation diagram

All the modes are clearly identified out from this diagram. Problems only occur for the first flexible mode, where the estimation of frequency gives

- 317 Hz for the narrow speed variation
- 348 for the full range run up/down

7. Conclusion

7.1 Results

Table 2 shows a comparison between frequency and damping of the modes identified by the input output modal analysis and the Operational Modal Analysis.

In the Operational Modal Analysis only the estimations from the SSI are tabled. The reason is that damping were only calculated for this estimator, although it is possible to estimate damping also by the FDD technique.

The problem for many mechanical applications is that rotating parts are often a main source to the vibrations. As this excitation is deterministic of nature a problem may arise that the full frequency range and thereby not all modes are excited.

Another problem is to separate forced sinusoidal components from peaks caused by structural resonances.

The measurements show that under the controlled environment in this test case it is possible to extract all modes in the frequency range.

For the first mode a deviation of the frequency is found using the narrow speed variation, but it was already seen from the contour plots that this mode was only exposed to an excitation in a very short period of time during the measurement.

One mode was not stable but indicated both by the stabilisation diagram as well as the FDD estimator.

All modes were identified and did in general show excellent relationship with those identified by input output modal analysis. Care must however be taken when the frequency range is not fully covered by the harmonics of the excitation forces. For identification of this problem the contour plots as achieved by the PULSE multianalyser are valuable tools.

7.2 General

Operational Modal Analysis has many advantages over input output Modal Analysis but care must be taken when assessing the data. In conjunction with other tools as FE Modelling Operational Modal Analysis is a strong tool for validating the model.

This paper has shown that also in connection with rotating machinery investigation Operational Modal Analysis has its advantage as it share the same acquired data and contour- or waterfall plots often acquired during investigations of rotating machinery are reused for validation of the Operational Modal Analysis estimations.

Input/Output Modal Analysis		Operational Modal Analysis			
		RPM Range 702 – 6000 RPM		RPM Range 5520 to 6060 RPM	
Frequency Hz	Damping %	Frequency Hz	Damping %	Frequency Hz	Damping %
351	1.38	348	2.17	317	1.65
478	0.51	479	0.33	478	0.59
698	0.76	701	0.59	704	0.49
841	0.42	848	0.29	847	0.26
945	0.73	951	0.39	947 (not stable)	0.34
1390	0.69	1394	0.56	1392	0.46
1619	0.71	1622	0.36	1631	0.59
1868	0.80	1865	0.40	1865	0.40
2117	0.81	2118	0.58	2117	0.34
2553	0.78	2555	0.63	2558	0.65
2768	1.23	2734	0.96	2742	0.98
3062	1.03	3059	0.74	3065	0.76

Table 2 Comparison between Operational Modal Analysis and Input output results

Nomenclature

Δt	sampling time step
y_t	response vector
f	natural frequency
ζ	damping ratio
Φ, Ψ	mode shape matrices
$MAC(i, j)$	MAC matrix

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