IDENTIFICATION TECHNIQUES FOR OPERATIONAL MODAL ANALYSIS – AN OVERVIEW AND PRACTICAL EXPERIENCES

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Abstract

Operational Modal Analysis, also known as Output Only Modal Analysis, has for several years been used for extracting modal parameters of mechanical structures. In this paper, an overview of the Frequency Domain Decomposition (FDD) technique and the Stochastic Subspace Identification (SSI) technique is given. Use of the recently developed projection channel technique in combination with the SSI technique is described and discussed.

Practical experiences in the use of these techniques are illustrated from measurement examples by comparing the results from the different techniques.

1 Introduction

Operational Modal Analysis is a technique for extraction of the modal parameters from vibration response signals. A main difference compared to the traditional mobility based modal analysis technique is that measurement of the input forces is not required. This enables testing of structures under operating conditions or in other situations were the input forces are impossible to measure. It is therefore also called Ambient Modal or Output only Modal. The technique has been known for a long time and the method has been used for civil engineering for more than a decade and in recent years within rotating machinery, automotive and aerospace applications.

Performing modal test under operating (ambient or natural) conditions means that the structure is subjected to realistic vibration behaviour, which might be difficult to obtain by use of artificial excitation. It also means that the test can be performed simultaneously with other response tests and it provides the possibility for extraction of modal information under conditions where a traditional mobility based modal test is very difficult or impossible to perform.

The measured responses are governed by the dynamic characteristics of the system and the forces, which excite the system. The derived model thus contains information of both the system characteristics as well as the excitation signals. This is one of the challenges in Operation Modal Analysis and some understanding of the nature and the characteristics of the excitation forces are therefore very important in order to interpret and understand the results and be able to derive a proper modal model.

This paper gives an overview of the practical use of the most commonly used operational modal analysis techniques, the Frequency Domain Decomposition (FDD) technique and the Stochastic Subspace Identification (SSI) techniques and application of the recently developed projection channel technique is covered as well.

Most of the practical aspects, comments and experiences are illustrated via the analysis of three different test objects:

a) A 1:5 scale model of a wind turbine wing (Fig.1a). It is a detailed model of one of the blades from a 675 kW wind turbine. The wing was made for lab investigations of static as well as dynamic parameters. Fig.1a shows a picture of the setup used for the measurements. The wing itself is supported by a console which is regarded as stiff compared to the wing itself. 24 accelerometers are mounted in two rows along the wing. Two time recordings were taken, one with the accelerometers perpendicular to the surface (Z-direction) and one pointing in the direction of rotation (X-direction). The X- and Z-directions of an additional point near one of the corners are used as reference DOF's (a DOF means a point and an associated direction) and the model is determined by combining (linking) the results from the two recordings together. The wing is considered as stiff in the length direction and vibrations in this direction are disregarded. The wing is exposed to an acoustic load by means of a loudspeaker placed beneath the wing. A microphone placed in front of the wing was used to measure the sound level spectrum in order to validate presence of energy in the frequency range of interest. See Refs. [1], [2] and [8] for more information on the setup, measurements and analysis results.

b) Outer part of an airplane wing (Fig.1b). Fig.1b shows a picture of the setup used for the measurements. The outer part of the wing is mounted on a support structure. 36 accelerometers are mounted on the surface giving measurements on 36 points perpendicular to the surface (Z-direction) in one measurement (one data set). Different experiments were performed using moving random impact excitation and acoustic excitation.

c) Plate with heavily coupled modes (almost repeated roots) (Fig.1c). Fig.1c shows a setup for measurements on a plate, featuring heavy coupling between the first two modes, the first bending mode and the first torsion mode (almost repeated roots). In one set of tests 12 accelerometers are used to measure 12 points perpendicular to the surface (Z-direction) in one measurement (one data set) and in another set of tests 5 accelerometers are used for measuring the 12 points in 4 measurements (4 data sets) by roving 3 accelerometers and having two accelerometers fixed reference DOF's. Different excitation methods are used, including single (fixed) broadband, dual (fixed) broadband and dual moving random impact excitation. A motor with rotating shaft is used as well in some of the tests.

***Fig.1a** Model of wind turbine wing analyzed using acoustical excitation*

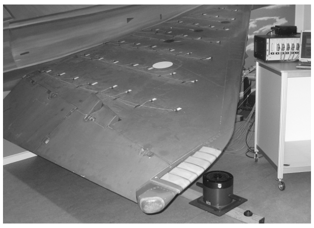


***Fig.1b** Outer part of an airplane wing*

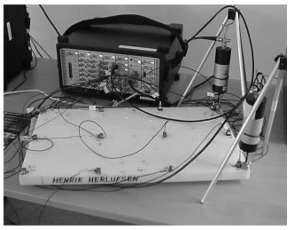


***Fig.1c** Plate with heavily coupled modes*

Traditional mobility based modal analysis is performed on the structures as well for comparison of the measurement results.

2 Data acquisition and validation equipment

For acquisition and validation of the response data a Brüel & Kjær PULSE multianalyser system is used together with Brüel & Kjær modal accelerometers. For some of the tests the handheld exciter Brüel & Kjær Type 5961 is used. PULSE multianalyser performs analysis and validation of the acquired time data in terms of contour plots of Short Time Fourier Transforms. This reveals the spectral distribution of the response signals as a function of time and content of sinusoidal frequency components can be detected. The frequencies of the main participating modes can often be identified from these contour plots. Fig.2 shows an example of a contour plot of a STFT of one of the response signals from a test of the plate. Apart from the broadband random content a number of sinusoidal components are clearly seen as well. These components are due to excitation forces from a motor running at almost constant speed. The speed was approximately 5850RPM, corresponding to a fundamental frequency of approximately 97,5Hz. The first harmonic as well as the third and fourth harmonic is present. The responses thus contain stationary operating deflection shapes at these frequencies (spectral ODS), which is very important information for the subsequent operational modal analysis.

The geometry and the time data are subsequently transferred into the Brüel & Kjær Operational Modal Analysis software for further signal processing and modal parameter extraction.

3 Modal parameter extraction methods

3.1 Signal Processing

In order to optimize the subsequent modal parameter extraction, by use of the time domain techniques, digital processing in terms of low-pass, high-pass, band-pass, band-rejection filtering and further decimation of the data can be performed. Possible requirement of filtering of the data depends upon the spectral distribution of the response signals. The first step of the analysis is therefore to calculate the Power Spectral Densities of the response signals and validate these together with the Short Time Fourier Transform (STFT) analysis performed earlier in the data acquisition process as described above and exemplified in Fig.2.

If for example the response signals have high content at low frequencies, due to high excitation of rigid body modes or measurement noise, a high-pass filtering of the signals can make the identification of the lower elastic modes, using the SSI technique, much easier. The filtering should, however, be made as “gentle” as possible, meaning that the order of the filter (giving the slope of the filter characteristic) should be as low as possible.

3.2 Frequency Domain Decomposition theory background

The Frequency Domain Decomposition (FDD) is an extension of the Basic Frequency Domain (BFD) technique, or more often called the Peak-Picking technique. This approach uses the fact that modes can be estimated from the spectral densities calculated in the condition of a white noise input and a lightly damped structure (Refs. [3], [4] and [8]). The FDD technique estimates the modes using a Singular Value Decomposition (SVD) of each of the Spectral Density matrices. This decomposition corresponds to a Single Degree of Freedom (SDOF) identification of the system for each singular value. In the following the most important relationships for understanding the FDD technique are given.

The relationship between the input x(t), and the output y(t) of a linear system can be written in the following form (Refs. [5] and [6])

$$\left[G_{yy}(\omega)\right] = \left[H(\omega)\right]^{*} \left[G_{xx}(\omega)\right]\left[H(\omega)\right]^{T}, \tag{1}$$

where $[G_{xx}(\omega)]$ is the input spectral matrix, $[G_{yy}(\omega)]$ is the output spectrum matrix, and $[H(\omega)]$ is the Frequency Response Function (FRF) matrix.

Writing the FRF matrix in the typical partial fraction form (used in classical Modal analysis), in terms of poles, λ and residues, R, and assuming that the input is random in both time and space, has a zero mean white noise distribution (i.e. $G_{xx}(\omega)$ = Const. for all the inputs) and that the damping is light, the response spectrum matrix can be written as the following final form (see Ref. [3]):

$$\left[G_{yy}(\omega)\right] = \sum_{k \in Sub(\omega)} \frac{d_k \boldsymbol{\psi}_k \boldsymbol{\psi}_k^{T}}{j\omega - \lambda_k} + \frac{d_k^{*} \boldsymbol{\psi}_k^{*} \boldsymbol{\psi}_k^{*T}}{j\omega - \lambda_k^{*}} \tag{2}$$

where $k \in Sub(\omega)$ is the set of modes that contribute at the particular frequency, $\boldsymbol{\psi}_k$ is the mode shape vector and d_k is a scaling factor for the k[th] mode. $\lambda_k = -\sigma_k + j\,\omega_{dk}$ is the pole of the k[th] mode, where σ_k is the modal damping (decay constant) and ω_{dk} the damped natural frequency of the k[th] mode. Eq. (2) expresses the response spectral matrix in terms of the modal parameters, λ_k and $\boldsymbol{\psi}_k$ and the scaling factor d_k , which is governed by the excitation.

Another way to understand the response signals is from their decomposition into participations from the different modes $[\Phi]$ expressed via the modal coordinates $\mathbf{q}(t)$:

$$\mathbf{y}(t) = [\Phi]\mathbf{q}(t) \tag{3}$$

Using eq. (3) in the expression of the correlation matrix of the responses we get:

$$[C_{yy}(\tau)] = E\{\mathbf{y}(t+\tau)\mathbf{y}(t)^{T}\} = E\{[\Phi]\mathbf{q}(t+\tau)\mathbf{q}(t)^{H}[\Phi]^{H}\} = [\Phi][C_{qq}(\tau)]\,[\Phi]^{H} \tag{4}$$

Applying the Fourier transform in eq. 4 gives:

$$[G_{yy}(\omega)] = [\Phi][G_{qq}(\omega)]\,[\Phi]^{H} \tag{5}$$

where $[G_{qq}(\omega)]$ is the spectrum matrix of the modal coordinates.

The FDD technique is based upon the SVD of the Hermetian response spectrum matrix at each frequency and for each measurement (data set):

$$\left[G_{yy}(\omega)\right] = [V]\,[S]\,[V]^{H} \tag{6}$$

where $[S]$ is the singular value diagonal matrix and $[V]$ is the orthogonal matrix of the singular vectors. The singular vectors (the columns in $[V]$) are orthogonal to each.

Eq. (6) has the same form as eq. (5) and it can be understood that the singular vectors present estimations of the mode shapes and the corresponding singular values present the response of each of the modes (SDOF systems) expressed by the spectrum of each modal coordinate The assumptions are that $[G_{qq}(\omega)]$ is a diagonal matrix, i.e. the modal coordinates are uncorrelated, and that the mode shapes (the columns in $[\Phi]$) are orthogonal.

Uncorrelated modal coordinates can be obtained by having uncorrelated random excitation forces with a random distributed over the structure (i.e. an excitation which is random in time and space).

A mode shapes should be estimated as close as possible to the corresponding resonance peak, where the influence of the other modes is as small as possible and the singular vector is most likely to give the best estimate of mode shape. In case of closely coupled modes, where the response is given by more singular values (response from more modes), the measurement DOF´s should be distributed over the structure such that the mode shapes are orthogonal and therefore likely to be estimated by the orthogonal singular vectors.

The simple peak picking technique gives frequency and associated mode shape at the selected frequency. The peaks in the SVD plot should be used as explained above.

In the later developed so-called Enhanced Frequency Domain Decomposition (EFDD) a SDOF model is imposed on the singular values in a user-defined frequency band around the peak providing the estimate of frequency and damping. An average of the corresponding singular vectors, weighted by the singular values in the band, provides the estimate of the mode shape. See Ref. [8] for a detailed description of the EFDD methods.

3.3 Stochastic Subspace Identification (SSI) theory background

The Stochastic Subspace Identification (SSI) techniques fit parametric models directly to the measured time responses. They are based upon the stochastic state space model described by:

$$\left.\begin{aligned} \mathbf{x}_{t+1} &= [\mathrm{A}]\mathbf{x}_t + \mathbf{w}_t \\ \mathbf{y}_t &= [\mathrm{C}]\mathbf{x}_t + \mathbf{v}_t \end{aligned}\right\} \tag{7}$$

where $\mathbf{x}_t$ is the state vector at time t, $[\mathrm{A}]$ is the system matrix (state matrix), $\mathbf{y}_t$ is the response vector at time t, and $[\mathrm{C}]$ is the observation matrix. The response is generated by two stochastic processes $\mathbf{w}_t$ and $\mathbf{v}_t$ called the process noise and the measurement noise respectively.

The steps in the SSI techniques from the time responses $\mathbf{y}_t$, via optimal predictors of $\mathbf{x}_t$, least square error estimates of $[\mathrm{A}]$ and $[\mathrm{C}]$ etc., to the estimated modal parameters are described in several references, including Refs. [6], [9], [10] and [11].

Modal models are estimated for the different state space dimensions up to a selected maximum state space dimension. The setting of maximum state space dimension depends upon the number of modes, which is searched for, the excitation, the number of sinusoidal components in the response signals and the number of noise modes needed to fit (predict) the measured response signals.

The results are achieved by a singular value decomposition of the full observation matrix, which is a matrix calculated from the measured responses, and extracting a subspace holding the modes in the model. Three different algorithms are often used in the SSI techniques, the *Unweighted Principal Component (UPC)*, the *Principal Component (PC)* and the *Canonical Variate Analysis (CVA)* algorithms.

A stabilization diagram for the modal models is used for selecting a model (at a certain state space dimension). Responses predicted from the models are compared with the measured responses in order to validate the selected model. The normalized singular values of the weighted observation matrix (or weighted common SSI input matrix) indicate the rank of the matrix on a scale from 0 to 1 and this value can therefore also be used as a guideline for the state space dimension required for the modelling. Is it difficult, however, to say how low this number should be and it is different for the three algorithms.

If the responses are measured in a sequence of measurements (data sets), a number of reference Degrees of Freedom (DOF's) must be included in each measurement (data set) and the models from each measurement are linked together afterwards.

Channel projection

In the case where a large number of response DOF's are measured simultaneously (i.e. measurement setups with large channel counts) the parametric model fit suffers from the estimation of many noise modes, compared to the number of physical modes of the system. The main reason for this is that the many channels contain the same physical information but different random errors. A way to reduce the amount of noise modes is therefore to reduce the number of channels in the actual estimation process. The information of the physical modes must not be affected and the selected channels must represent the system.

A simple measure of the amount of information of a measurement channel compared to the other channels can be established from calculation of the correlation coefficients between the different measurement channels

The first step is to find the channel that correlates most with all the other channels. This channel most likely contains maximum physical information

The second step is to find the remaining number of requested projection channels. These are found by similar search of the correlation coefficient matrix, as channels that correlate the least with all previously found projection channels. These channels will most likely bring maximum of new information. The only pitfall here is if a channel is dead and only contains noise. In such a case it will have an insignificant correlation with the other channels. To prevent this lower threshold of allowed correlation should be applied.

In case of multiple data sets the first step above is excluded. Instead the user-defined reference channels are applied as initial projection channels. The assumption is that all the modes are sufficiently present in the reference DOF´s. The remaining step is as described above.

Use of projection channels decreases the amount of redundant information and the estimated models tends to stabilize faster, i.e. at lower state space dimensions (corresponding to "smaller" subspaces). In addition, computation time is decreased since the matrix operations are simplified significantly when having many measurement channels.

Choice of number of projection channels will be discussed below in the examples.

4 Overview of analysis results and experiences

4.1 Frequency Domain Decomposition (FDD) method

The FFD technique is the most simple and straightforward method to use. After validation of the STFT and the averaged spectra of the response signals, the modes are selected from the plot of the SVD of the response spectral density matrix. If there is more than one measurement (more than one data set) it is the average of the Singular Values of the spectral densities of each of the measurements, which is used for the selection of the peaks.

Fig.3a shows the average of the SVD (FDD) of the two measurements of the responses from the wind turbine wing model (Fig.1a) in a test using acoustical excitation (low frequency random).

A number of peaks appear very clearly in the SVD, specially in the low frequency range below 100Hz. These peaks are expected to be caused by structural resonances and not by response due to a high level force excitation in a narrow-band. 17 modes are detected below 110Hz (Fig.3a). The frequency and damping values are determined using a SDOF model applied in a user-definable frequency band around the peak and the mode shapes are determined from the singular vectors weighted by the singular values in the frequency band. All the resonances are well separated in the plot even the two lowest modes at 7,8Hz and 8,6Hz due to the relative low damping (approximately 1,4%).

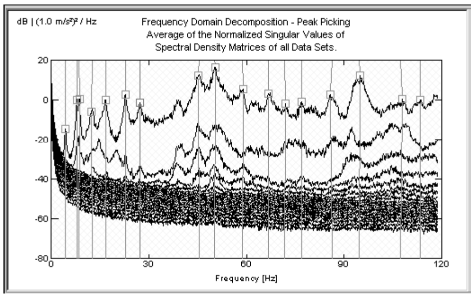


Fig.3a *SVD plot from the test of the wind turbine wing model*

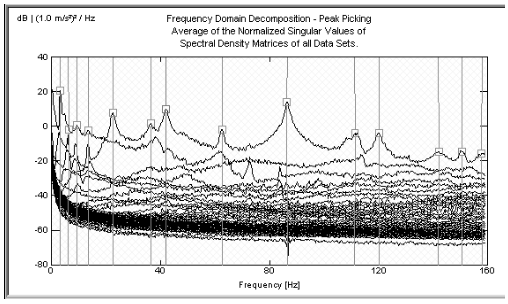


Fig.3b *SVD plot from a test of the outer part of an airplane wing*

A SVD plot from a test of the outer part of an airplane wing using moving random impact is shown in Fig.3b. Another example where a number of separated peaks, caused by structural resonances, are clearly seen allowing for modal model estimations using the EFDD technique.

The underlying assumption for the operational modal identification techniques is that the excitation is from broadband random forces distributed over the structure (random in time and space). The SVD plot is very useful to validate whether this is the case or not. The use of the FDD technique and the interpretation of the SVD plot are illustrated in the following via some simple tests using different excitations of the system.

Figs.4a, 4b, 4c and 4d shows the SVD plots from four different tests of the plate (Fig.1c) featuring heavy coupling between the first bending and the first torsion modes at 187Hz and 189Hz. The damping ratio of the modes is approximately 3,5%, which means that the 3dB bandwidth for each mode would be approximately 13Hz if it was isolated from the other mode as a SDOF system. Heavily coupling, or almost repeated roots, means that the 3dB bandwidth of the individual resonances is much larger than the difference between the resonance frequencies. These examples illustrate the importance of proper excitation of the system in order to be able to estimate the model. In Fig.4a a single broadband source is exciting the plate in a corner point and it is seen that the SVD plot is dominated by only the first singular value. The first singular value (green curve) is much higher than the second singular value (red curve) and the estimates of the two closely spaced modes at around 188Hz from the first and the second singular value (green and red curve, respectively) is very poor. Both frequency and damping values are biased and the mode shapes appear complex and as linear combinations of the two shapes.

In Fig.4b two broadband sources excite the plate in two corner points, from which the two modes are different (mode shapes at the two points are in-phase for bending and out-of-phase for torsion). The SVD plot is in this case dominated by the first two singular values corresponding to the two independent sources and the modal parameters for the two coupled modes can be estimated correctly, see Fig.5a and 5b, with results that are in agreement with the results from classical mobility based poly-reference technique. Notice that the bending mode (186,6Hz with 3,4% damping) is determined from the SDOF resonance curve in the second singular value and the torsion mode (188,9Hz with 3,7% damping) from the SDOF resonance curve in the first singular value.

In order to illustrate the importance of proper distributed excitation Fig.4c shows the SVD plot in the case of two broadband sources exciting the plate as in Fig.4b, however in two corner points diagonal to each other, from which the two modes look the same (mode shape of the two points are in-phase for bending as well as for torsion). In this case the second of the two modes can only be seen in the third singular value with biased estimates of both frequency and damping and very poor estimate of the mode shape as in the case of only one excitation source (Fig.4a). This corresponds to the situation of separating closely spaced modes using classical mobility based poly-reference technique. Separation of the two modes requires that the mode shapes are different (orthogonal) at the reference DOF's.

In Fig.4d the plate is excited by two broadband sources moving around on the surface and the SVD plot shows much higher levels in the lower singular values. Notice a much clearer appearance of the two modes at 495Hz and 520Hz in the first singular value and their modal parameters can be extracted in better agreement with the results from classical mobility based measurements.

Content of sinusoidal components can be identified from contour plots of the STFT as illustrated in Fig.2. Fig.6 shows a response autospectrum (top) and a SVD plot (bottom) from a similar test with an excitation signal having broadband random content together with a number of sinusoidal components (harmonics) from a motor running at constant speed. With the selected frequency resolution (1Hz spacing between the FFT lines) the harmonic components above 300Hz are buried in the power spectral density of the random signal and can therefore not be seen in the autospectrum. In the SVD plot, however, the harmonic components appear very clearly in the lower singular values. The spectral components influencing the highest singular values can influence the modal parameter extraction from the broadband random response. The operating deflection shapes at the discrete frequencies (spectral ODS) must be known in order to be able to distinguish between these and modes in the validation of the extracted modal models. A harmonic component is seen at 195Hz, i.e. very close to the frequencies of the bending and torsion modes and this obstructs the identification of these modes. This sinusoidal component is within the 3dB bandwidth of the modes and proper identification of the modes is very difficult even after careful ("gentle") band-rejection filtering of the component.

The SVD plot might also reveal poor signal quality if not already revealed in the validation (STFT) or the signal processing (power spectral densities) as discussed above. Poor signal quality could be due to improper signal conditioning or a damaged transducer or cable. Fig.7 shows the SVD plot of a measurement similar to the one in Fig.4d, however with one of the transducers disconnected, causing the low level (noise) singular value.

The SVD plots should therefore always be investigated no matter which method is used for the modal parameter extraction.

4.2 Stochastic Subspace Identification (SSI) method

In Refs. [1], [2], [8] and [9] analysis of the wind turbine wing model by use of EFDD and SSI techniques as well as classical mobility based method is discussed. The three SSI methods and EFDD gave similar results, with respect to frequencies and mode shapes. In the Refs. [1] and [2] the modes below 40Hz were obtained using the SSI techniques after low-pass filtering and decimation to 50Hz. These analyses were performed before the use of projection channels was developed. The two measurements (two data sets) contain 26 DOF's each, including the two reference DOF's, and use of projection channels is an obvious choice. The SVD plot in Fig.3a shows that the information in the response signals is carried in the first three to four singular values indicating that an optimum number of projection channels should be between three and five.

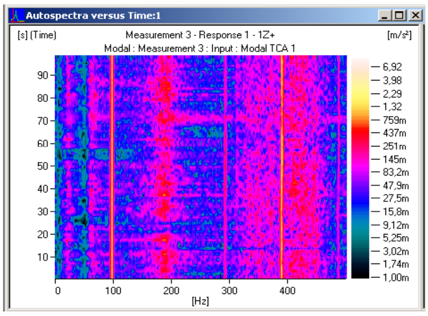
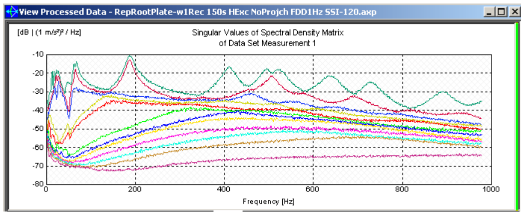
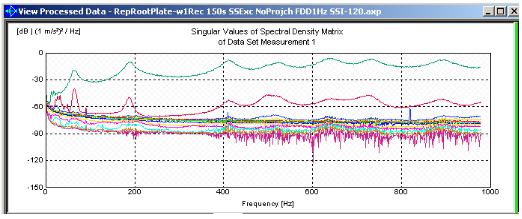


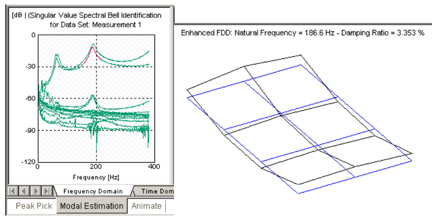
Fig.2 *Example of a contour plot of a STFT of a response signal revealing random as well as sinusoidal content*



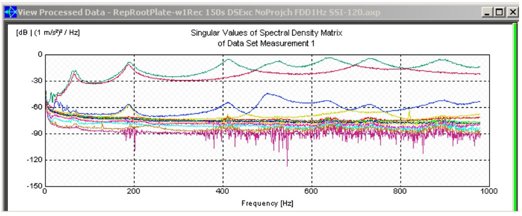
Figs.4d *SVD with two moving broadbanded excitation sources*



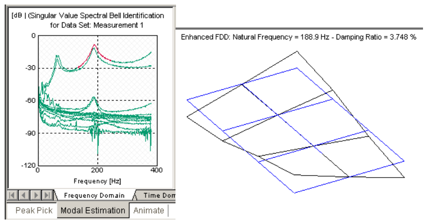
Figs.4a *SVD with one fixed broadbanded excitation source*



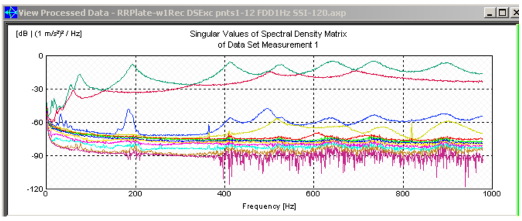
Figs.5a *Bending mode estimated from the FDD (SVD) shown in Fig 4b*



Figs.4b *SVD with two fixed broadband excitation sources*



Figs.5b *Torsion mode estimated from the FDD (SVD) shown in Fig 4b*



Figs.4c *SVD with two fixed broadband excitation sources (different points than in 4b)*

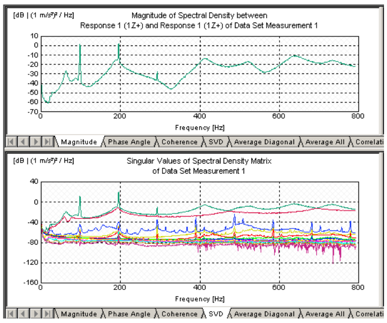
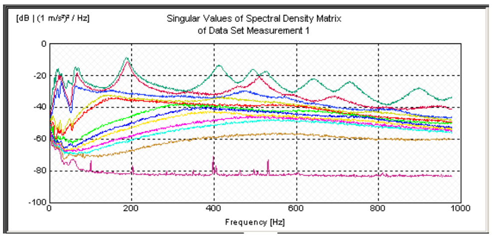


Fig.6 *Autospectrum and SVD plot from a test similar to the one shown in Fig.2*



Figs.7 *SVD with one transducer disconnected, otherwise as in Fig.4d*

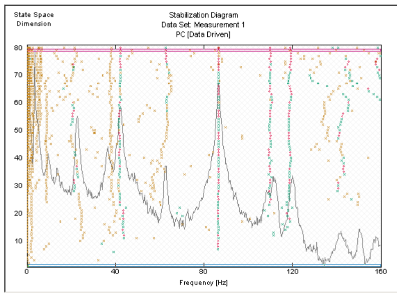


Fig.9a *Stabilization diagrams without projection channels*

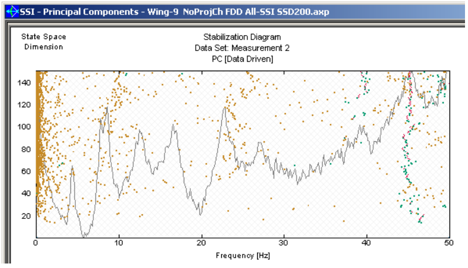


Fig.8a *No projection channels*

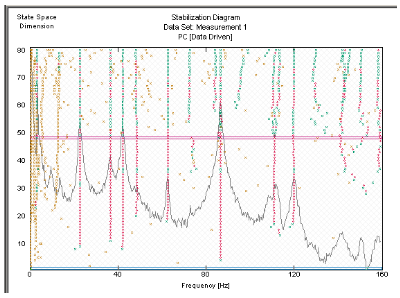


Fig.9b *Stabilization diagrams with five projection channels*

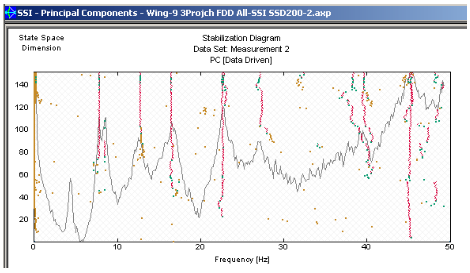
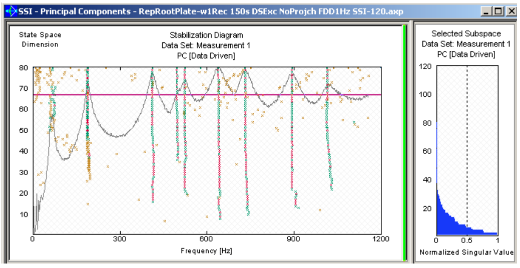


Fig.8b *Three projection channels*



Figs.10a *Stabilization diagram with two fixed broadbanded excitation sources*

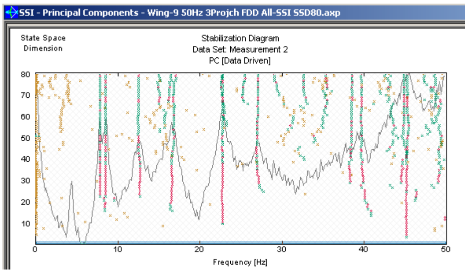
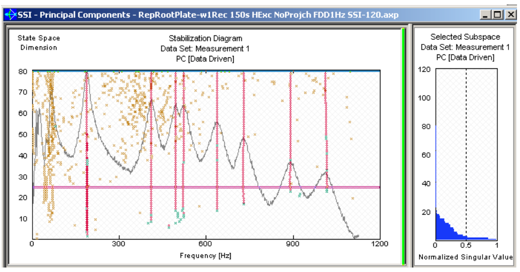


Fig.8c *Decimation to 50Hz and three projection channels*



Figs.10b *Stabilization diagram with two moving broadbanded excitation sources*

Figs.8a, 8b and 8c shows the stabilization diagrams for the PC algorithm in the frequency band up to 50Hz for different analyses of the second data set. In Fig.8a the analysis is performed without projection channels, with a frequency range of 200Hz and a maximum state space dimension of 200. The stabilization diagram is shown up to state space dimension 150 and none of the modes below 40Hz have been identified. In Figs.8b three projection channels are used with everything else unchanged compared to the analysis in Fig.8a. With three projection channels most of the lower modes are identified at a state space dimension of app. 80. The second mode at 8,5Hz, however, is not identified in this range of state space dimension. Analysis using four or five projection channels gives less stabilization of the modes indicating an optimal choice of three projection channels. In Figs.8c the time data has been low-pass filtered and decimated to 50Hz and three projection channels are used with a maximum state space dimension of 80. All modes below 50Hz are identified from a state space dimension of app. 30 and the variation of the estimated modal parameters between the two measurements is much smaller compared to the results from the analysis without decimation in Fig.8b. Using low-pass filtering and decimation to 50Hz, without projection channels, identification of the modes below 40Hz requires much higher state space dimensions with less stabilization and much more noise modes in the identification. The same tendency is seen for the UPC and the CVA algorithms. This illustrates the advantage of combining the use of projection channels with low-pass filtering and decimation in situations of higher number of channels and modal identifications over several octaves: The lower frequency modes can be identified with better stability and at lower state space dimensions with fewer noise modes.

The responses from the 36 points of the outer part of the airplane wing were measured simultaneously in one measurement (one data set). The SVD plot in Fig.3b shows that all the information in the 36 signals is carried by the first four to five singular values. Analysis was performed for all the three algorithms (UPC, PC and CVA) without projection channels and by use of three, five and eight projection channels. The stabilization diagrams for the PC algorithm, up to a state space dimension of 80, are shown without use of projection channels in Fig.9a and with use of five projection channels in Fig.9b. The analyses have a maximum state space dimension of 120 and a frequency range of 200Hz. Without projection channels there is very poor stabilization for the modes below 80Hz. Use of projection channels improves the identification and stabilization of the modes down to app. 20Hz. Five projection channels gives the best stabilization, better than three projection channels and slightly better than eight projection channels. This is in good agreement with the number of significant singular values (Fig.3b). All three SSI algorithms give almost identical results and they agree very well with those found by the EFDD method (Fig.3b) up to 150Hz.

The ability of the SSI technique to estimate closely coupled modes (almost repeated roots) is illustrated with the tests on the plate (Fig.1c). A number of different tests are performed as discussed in section 4.1. The importance of having proper excitation of the system is illustrated in Figs.10a and 10b. In Fig.10a two fixed broadband excitation sources are used (same test as in Fig.4b) and in Fig.10b two moving broadband excitation sources are exciting the system (same test as in Fig.4d). They show the stabilization diagram for the PC algorithm up to a state space dimension of 80. The maximum state space dimension is set to 120 and projection channels are not used. All the modes can be identified in both cases, including the closely coupled bending and torsion modes at app. 188Hz (see Figs.5a and 5b), but the stabilization is much faster and better in the case of two moving excitation sources and the mode shapes for the bending and the torsion modes are also better estimated in the case of moving excitation sources (more real and less coupled). The modal parameters for the third and the higher elastic modes are almost identical for the two cases, but much higher state space dimension (more than twice) is required in the case of the fixed excitation sources. This is also indicated in the normalized singular values of the weighted observation matrix shown to the right in Figs.10a and 10b, next to the stabilization diagrams (notice that the normalized singular values are shown up to the maximum state space dimension of 120). The state space dimensions selected in this analysis using the PC algorithm are 67 and 25 for the two fixed excitation sources and the moving excitation sources, respectively.

The UPC and CVA algorithms gave results almost identical to those given by the PC algorithm. The selected state space dimension follows the general trend, namely slightly higher for the UPC and somewhat higher for the CVA.

In the tests where the 12 DOF´s are measured simultaneously it is worth to investigate the use of projection channels. In the case of two fixed excitation sources, most of the information is carried in the three or four highest singular values (see Figs.4b and 4c). Five projection channels gives slightly faster stabilization with the same results except for the mode shapes of the closely coupled modes being slightly more complex. Three projection channels gives almost same stabilization as without projection channels, except for the mode at 411Hz which stabilizes slower. When this happens it is often because the (automatic) choice of projection channels is not optimal for that mode and this situation can be verified by investigating the SVD plot. In this case the mode at 411Hz appears in the second singular value when three projection channels is used and the stabilization in the SSI is slower. The conclusion is therefore that there is no need for use of projection channels in this case with only 12 measurement channels.

The SSI method for the tests on the plate gives mode shapes, for the third and the higher modes (411Hz and higher), which are more real (normal) that those given by use of the EFDD method. The SSI and the EFDD methods give similar results for the closely coupled bending and torsion modes in the various excitation situations.

5 Conclusion

An overview of the identification techniques for operational modal analysis is given and some experience in their practical use is illustrated from the analysis of a number of tests on three different test objects.

The Frequency Domain Decomposition (FDD) technique is the most simple and straightforward method to use when the SVD plot reveals isolated peaks caused by resonances. The simple peak picking technique gives frequency and mode shape at the selected frequency, whereas the Enhanced FDD (EFDD) method adds estimation of damping as well and the estimates of frequency and mode shape are improved by fitting a SDOF model to the singular values in a user-defined frequency band around the peak.

In the case of heavily coupled modes (almost repeated roots) the modal information is incorporated in more singular values in the frequency band around the resonances. The example shows that proper estimation of the modes requires an excitation, which is sufficiently distributed over the structure.

The SVD plot of the response signals is shown to be an excellent validation tool. It can reveal insufficient excitation of the system and poor quality of the signals and should therefore always be investigated no matter which method is used for the modal parameter extraction.

The different SSI algorithms (UPC, PC and CVA) give almost identical results. The general trend is that the required state space dimension is lowest for the PC, slightly higher for the UPC and highest for the CVA, as also indicated by the normalized singular values of the weighted observation matrix and shown together with the stabilization diagram.

In cases where a large number of response DOF's are measured simultaneously (i.e. measurement with large channel counts) use of projection channels is essential and for modal identifications over several octaves use of projection channels together with low-pass filtering and decimation gives much better and faster stabilization. Use of projection channels also reduces the calculation time. Indication of the number of projection channels required is given in the number of significant singular values in the SVD plot.

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