Output Only Modal Testing of a Car Body Subject to Engine Excitation

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Abstract

In this paper an output only modal testing and identification of a car body subject to engine excitation is presented. The response data were analysed using two different techniques: a non-parametric technique based on Frequency Domain Decomposition (FDD), and a parametric technique working on the raw data in time domain, a data driven Stochastic Subspace Identification (SSI) algorithm. Both techniques identified 16 modes under 85 Hz. The results of the two techniques were validated against each other. It was concluded, that even though some of the identified modes showed an unsatisfactory agreement when comparing results of the two techniques, most of the modes compared well, and thus, for the main part of the identified modes the modal estimates most be considered as reliable modal parameters.

Nomenclature

Δt	sampling time step
y_t	response vector
f	natural frequency
ς	damping ratio
Φ, Ψ	mode shape matrices
$MAC(i,j)$	MAC matrix

Introduction

When modal properties are to be identified using output only techniques, the modal identification has to be carried out without using information about the input exciting the structure. In this particular case, a part of a car body was exited by running the engine changing the RPM of the engine to perform a "sweep" over a suitable frequency range. The loading so encountered was not measured.

The response data were analysed using two different techniques: a non-parametric technique based on Frequency Domain Decomposition (FDD), and a parametric technique working on the raw data in time domain, a data driven Stochastic Subspace Identification (SSI) algorithm.

The results from the two techniques were compared and validated against each other.

Test conditions and data pre-processing

The data were measured during a test of the chassis vibration due to the load from the engine. The vibrations were measured on the chassis at the four mounting points during the run up and down of the engine. Three axial accelerometers were used on the points carrying at total 12 signals

The signals were acquired by means of a Brüel & Kjær PULSE™ multi-analyser system, where frequency analysis were done real time while full time recording takes place on a throughput disk. The reponse signals were sampled at a sampling rate of 1024 data points per second, and 56320 data points were acquired in each time series corresponding to a total measurement time of 55 seconds.

For the modal analysis the raw time data were exported into the ARTEMIS software by a dedicated export tool called PULSE bridge to MATLAB. This also gives the possibility of using data from PULSE in the MATLAB™ environment.

A time-frequency plot obtained using a traveling window FFT analysis on the original data is shown in figure 1. The engine sweep and the corresponding over harmonics are clearly visible in the signal.

The data were decimated by a factor 5 corresponding to a Nyquist frequency of 102.8 Hz, and the modal identification was carried out in the range 0-85 Hz.

Since the SSI technique has difficulties identifying as many modes as was present in this frequency range from 0-85 Hz, the data were digitally band-pass filtered in five frequency bands: 0-21 Hz, 21-41 Hz, 41-62 Hz, 62-82 Hz and 82-103 Hz.

Principle of Frequency Domain Decomposition (FDD)

The Frequency Domain Decomposition (FDD) technique is an extension of the classical frequency domain approach often referred to as the Basic Frequency Domain (BFD) technique, or the peak picking technique. The classical approach is based on simple signal processing using the Discrete Fourier Transform, and is using the fact, that well separated modes can be estimated directly from the power spectral density matrix at the peak.

I the FDD technique first the spectral matrix is formed from the measured outputs using simple signal processing by discrete Fourier Transform (DFT). However, instead of using the spectral density matrix directly like in the classical approach, the spectral matrix is decomposed at every frequency line using Singular Value Decomposition (SVD). By doing so the spectral matrix is decomposed into a set of auto spectral density functions, each corresponding to a single degree of freedom (SDOF) system. This is exactly true in the case where the loading is white noise, the structure is lightly damped, and where the mode shapes of close modes are geometrically orthogonal. If these assumptions are not satisfied, the decomposition into SDOF systems is an approximation, but still the results are significantly more accurate than the results of the classical approach.

The singular vectors in the SVD are used as estimates of the mode shape vectors, and the natural frequencies are estimated by taking each individual SDOF auto spectral

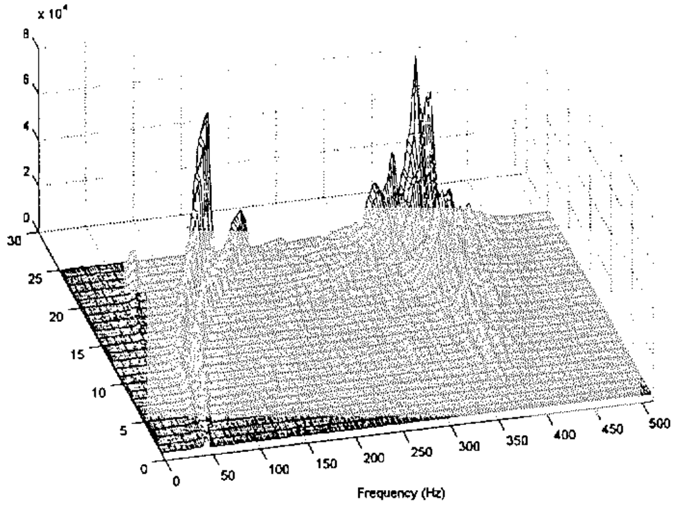


Figure 1. Time-frequency plot using a travelling window FFT analysis.

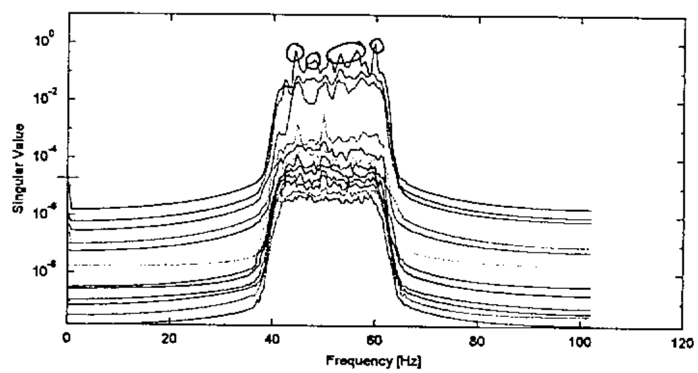
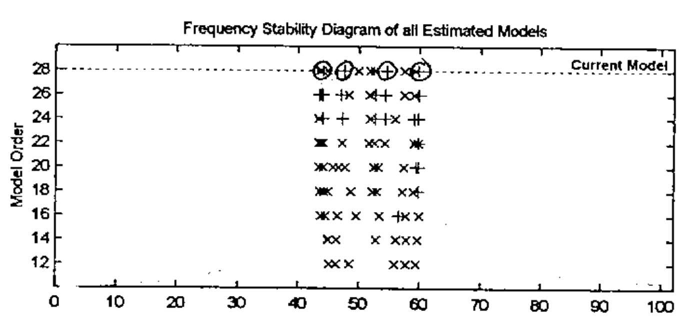


Figure 2. Singular value decomposition of the spectral density matrix of the signal band-pass filtered 41-62 Hz.



Figur 3. Stabilisation diagram of the signal band-pass filtered 41-62 Hz.

density function back to time domain by inverse DFT. The frequency and the damping were simply estimated from the crossing times and the logarithmic decrement of the corresponding SDOF auto correlation function.

The theoretical background of the FDD technique is described in Brincker et al [1].

Results of Frequency Domain Decomposition (FDD)

Figure 1 shows the singular value decomposition of the spectral density matrix for the frequency range 41-62 Hz.- up. Since the number of channels is equal top 12, 12 singular values are present

Taking this example, the FDD technique identifies the first peak from the left as one mode, the next peak is also identified as a mode, the next three peaks are identified as one mode, and finally the last peak from the left is identified as a mode. Three singular values are high and the rest are low. This means, that in this frequency range at any frequency line three modes are interfering.

Damping and natural frequencies are given in Table 1, and mode shapes for the lowest 6 modes are shown in Figures 4,5 and 6.

Principle of Stochastic Subspace Identification (SSI)

Stochastic Subspace Identification (SSI) is a class of techniques that are all formulated and solved using state space formulations of the form

$$x_{t+1} = Ax_t + w_t$$
$$y_t = Cx_t + v_t$$

where x_t is the Kalman sequences that in SSI is found by a so-called orthogonal projection technique, Overschee and De Moor [3]. Next step is to solve the regression problem for the matrices A and C, and for the residual sequences w_t and v_t. Finally, in order to complete a full covariance equivalent model in discrete time, the Kalman gain matrix K is estimated to yield

$$\hat{x}_{t+1} = A\hat{x}_t + Ke_t$$
$$y_t = C\hat{x}_t + e_t$$

It can be shown, Brincker and Andersen [2], that by performing a modal decomposition of the A matrix as $A = V[\mu_i]V^{-1}$ and introducing a new state vector $z_t = V^{-1}\hat{x}_t$ the equation can also be written as

$$z_{t+1} = [\mu_i]z_t + \Psi e_t$$
$$y_t = \Phi z_t + e_t$$

where $[\mu_i]$ is a diagonal matrix holding the discrete poles related to the continuos time poles λ_i by $\mu_i = \exp(\lambda_i \Delta t)$, and where the matrix Φ is holding the left hand mode shapes (physical, scaled mode shapes) and the matrix Ψ is holding the right hand mode shapes (non-physical mode shapes). The right hand mode shapes are also referred to as the initial modal amplitudes, Juang [4]

The specific technique used in this investigation is the Principal Component algorithm, see Overschee and De Moor [3].

Results of Stochastic Subspace Identification (SSI)

For each of the frequency ranges a set of models with different model orders were identified and the stabilisation diagram was established. Figure 3 shows the stabilisation diagram for the frequency range 41-62 Hz.

As it appears, four modes were stabilised in this frequency band. However, since a model order of 28 was needed to stabilise the modes, totally 14 modes are present in the chosen model. This means that in additional to the four stabilised modes 10 noise modes are present in the same frequency range. Some of these noise modes might in fact be physical modes. However, since we will only use the modes that can be identified by both techniques, the stabilisation criteria were adjusted to accept only four modes.

Damping and natural frequencies are given in Table 1, and mode shapes are shown in Figures 4, 5 and 6.

Validation of results

First of all, one most be sure that the identified modes represent something physical. In this case, since the excitation is non-stationary, and since the sweep of the harmonics created by the engine might stop and start inside the frequency range, computational modes might arise that could look physical, but is in fact just representing the non-stationary loading. To have confidence in the identified modes, the identification was carried out twice: one identification using all data points (the results presented here), and one identification using 90 % of the data points. All modes presented in this paper were present in both identifications.

The results of the two techniques were validated against each other. Natural frequencies, damping ratios can be compared directly and mode shapes can be compared by visual inspection of the mode shape plots. In this case however, since the mode shape is known in only four points, the spatial resolution is poor, and thus, a visual evaluation is difficult. However, inspecting the first 8 modes shown in Figure 4-6, it appears clearly, that significant deviations are present for some of the mode shape estimates of the two techniques. To evaluate the mode shapes the Modal Assurance Criterion matrix MAC was calculated, where $MAC(i,j)$ denotes the MAC value between mode i in FDD estimation with mode j in SSI estimation. The diagonal elements of the MAC matrix are given in table 1. A measure of modal significance was also calculated as

$$MSC = MAC(i,i)^2 / (MAC_{i\max}^{FDD} MAC_{i\max}^{SSI})$$

where $MAC_{i\max}^{FDD}$ is the maximum off-diagonal element in row no i and $MAC_{i\max}^{SSI}$ is the maximum off-diagonal element in column no i. Thus this measure compares the mode shapes in one model with the other mode shapes (other modes) in the other model. If the measure is larger than one, it means that the mode shape compares better with the corresponding mode in the other model than with any other mode shape in the other model.

The first two modes seem to be rather poorly identified. Damping is uncertain, some frequency deviation is present and the MAC value is low. However, the modal significance is high. This indicates, that even though the modes are uncertain, the identified mode shapes are really characteristic for these modes. Some of the modes have modal significance close to one (modes 8,9,11,12). This indicates, that for these modes several modes in the set of modes look similar. Considering the validation measures, the modes that should be considered uncertain are modes 5, 10, 13 and 16. The modes 8,9,11,12,14 should be considered as having some uncertainty, and the rest of the modes are to considered as reliable.

Generally damping is estimated with a high uncertainty , however, for modes 3 and 4, the damping is believed to be reliably estimated and having a relative high value (4-8 %) and for modes 7,11 also damping is believed to be reliably estimated, but in this case the damping is relative light (0.5-1 %)

Conclusions

Using less than one minute of response data from a car body excited by the engine 16 modes have been identified in the frequency range 0-85 Hz using two different output only identification techniques, the FDD technique and the SSI technique. All modes were identified by both techniques, and all modes were identified also in case of reducing the data. Thus, it is believed that all 16 modes represent physical modes of the car body.

Using different techniques to validate the mode shape estimates for the two techniques it has been indicated that some of the modal estimates are uncertain. However, most of the modes were indicated to be reliable modes and the corresponding modal parameters is to be considered as reliable modal parameters

Frequency deviations between natural frequencies of the estimates obtained by the two techniques were typically of the order of 0.5 Hz or 0.5 % of the Nyquist frequency.

The damping estimates were in the most cases quite uncertain. For a few modes however, damping seemed to be estimated accurately.

References

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[4] Juang, J.N.: "Applied System Identification", Prentice Hall, Englewood Cliffs, New Jersey, 1994.

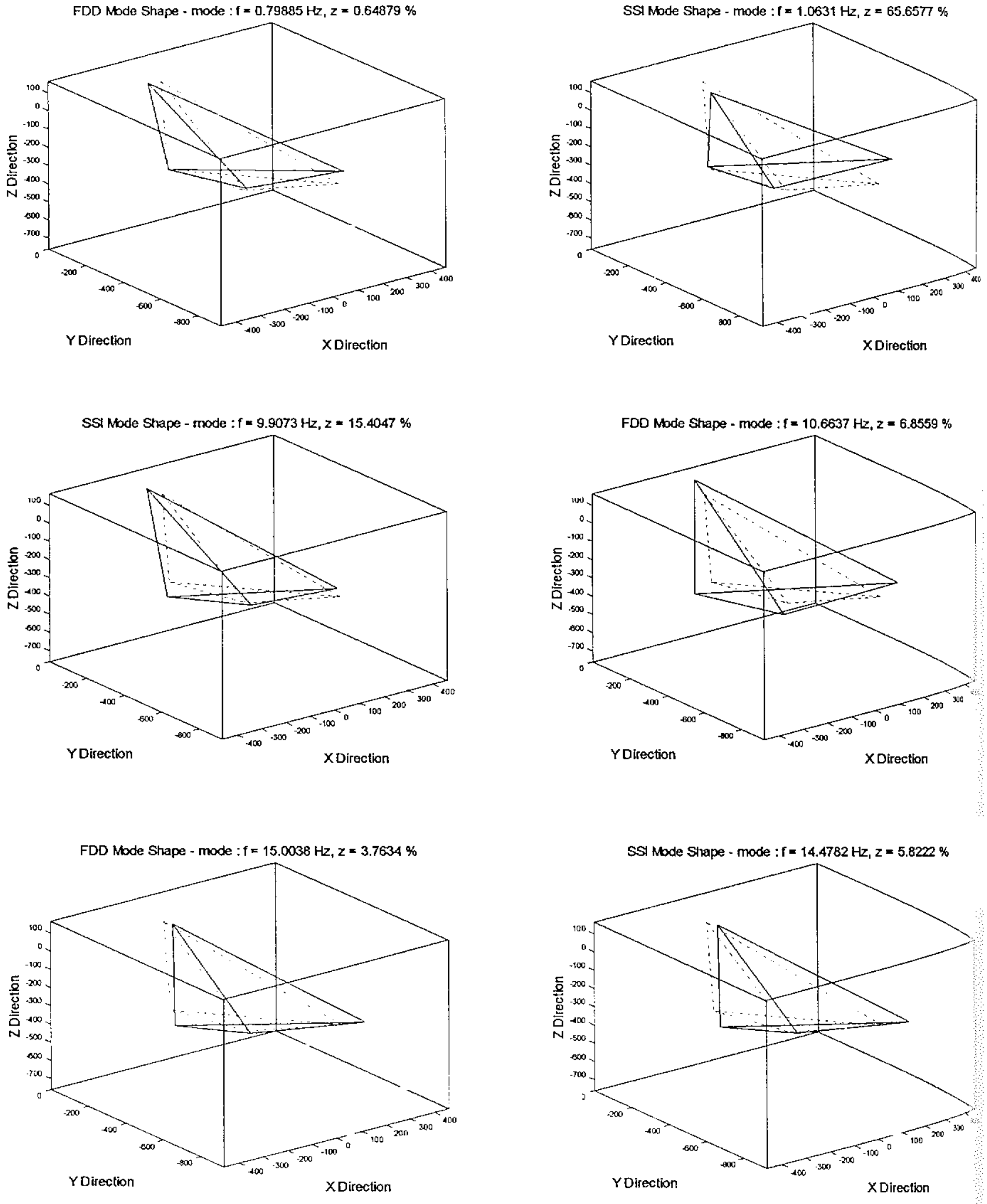

Figure 4. Mode 1-3, left: Results of Frequency Domain Decomposition (FDD), right: Results of Stochastic Subspace Identification (SSI)

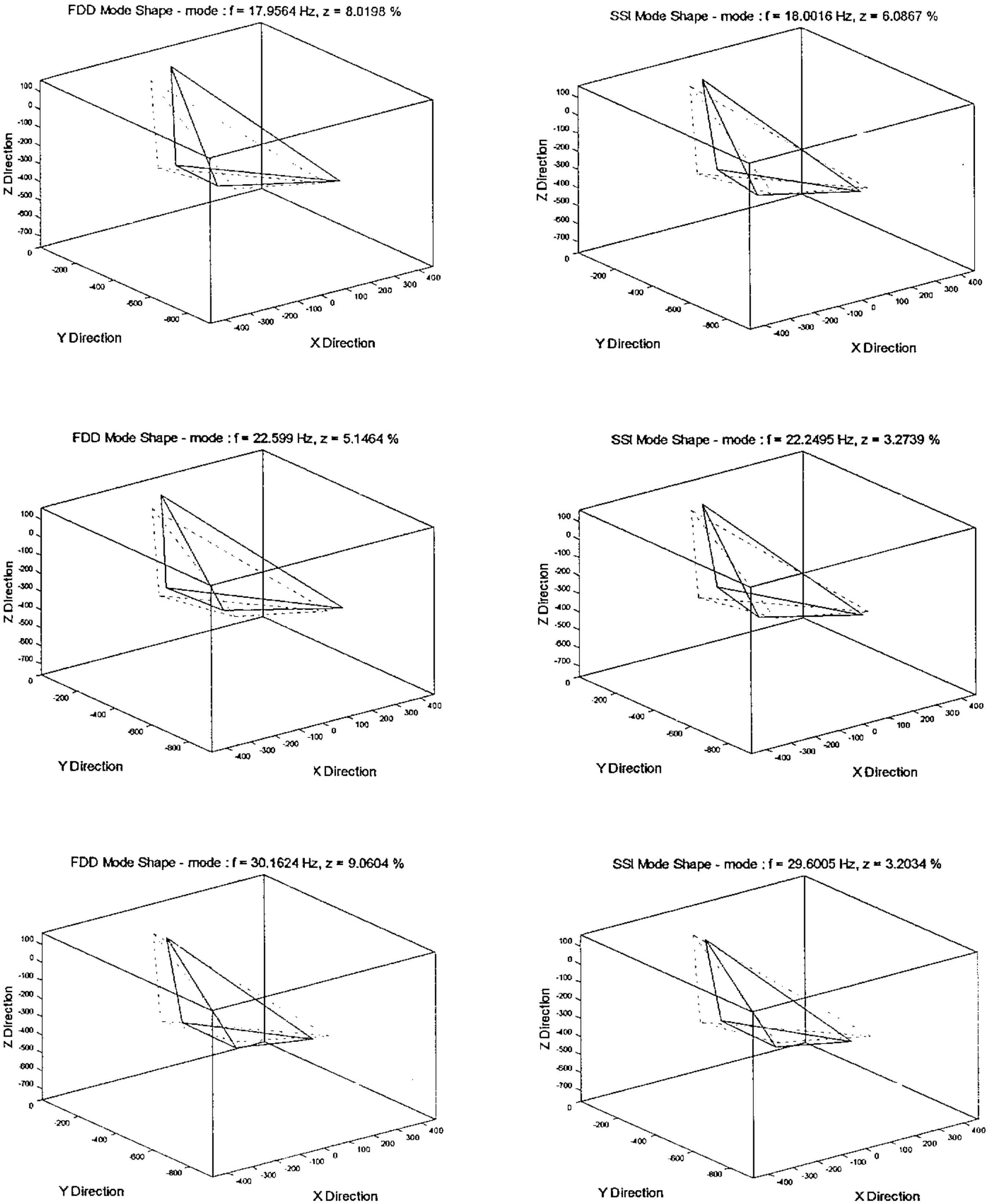

Figure 5. Mode 4-6, left: Results of Frequency Domain Decomposition (FDD), right: Results of Stochastic Subspace Identification (SSI)

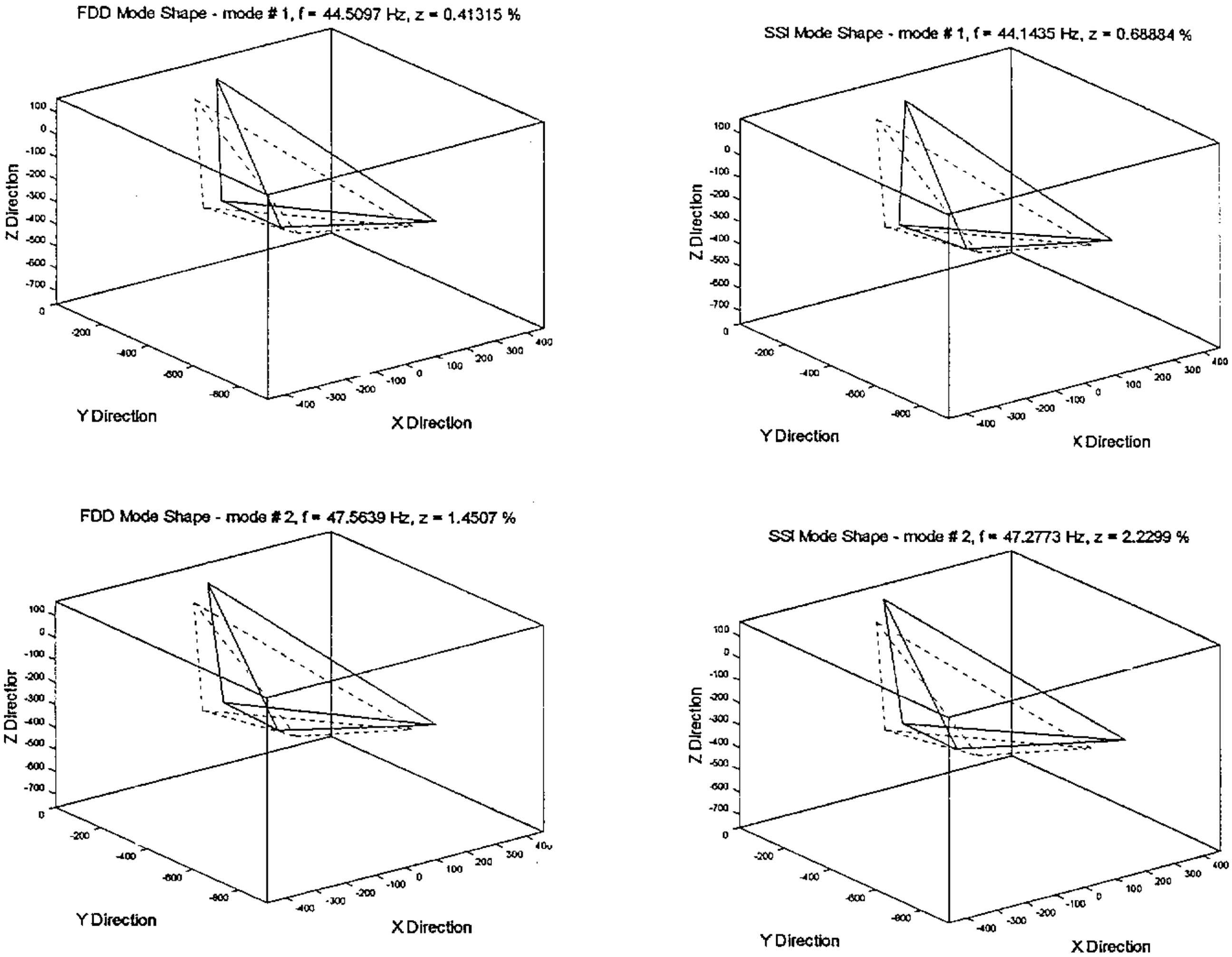

Figure 6. Mode 7-8, left: Results of Frequency Domain Decomposition (FDD), right: Results of Stochastic Subspace Identification (SSI)

Table 1. Results of the output only modal identification.

Mode No.	Frequency Domain Decomposition (FDD)		Stochastic Subspace Identification (SSI)		Modal Assurance (MAC)	Modal Signifi-cance
	Frequency (Hz)	Damping (%)	Frequency (Hz)	Damping (%)		
1	0.80	0.65	1.06	65.66	0.858	2.47
2	9.91	15.40	10.66	6.86	0.624	2.40
3	15.00	3.76	14.48	5.82	0.947	2.09
4	17.95	8.02	18.00	6.09	0.852	1.10
5	22.60	5.15	22.25	3.27	0.718	0.68
6	30.16	9.06	29.60	3.20	0.989	1.21
7	44.51	0.41	44.14	0.69	0.987	1.07
8	47.56	1.45	47.27	2.22	0.976	1.01
9	54.40	1.01	54.28	2.81	0.981	1.01
10	59.91	0.36	59.13	1.38	0.275	0.11
11	62.54	0.67	62.91	0.85	0.984	1.01
12	66.52	0.72	67.40	2.26	0.980	1.00
13	72.11	0.97	71.62	1.83	0.239	0.06
14	74.98	0.50	74.71	0.83	0.978	0.99
15	81.52	0.84	79.68	1.91	0.811	0.88
16	85.06	1.40	84.15	1.59	0.558	0.41