Kalman filter-based stochastic subspace identification under mixed stochastic and periodic excitation

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CONTEXT
- Subspace-based system identification from output-only vibration measurements collected from structures in-operation
- Modal analysis of civil, mechanical or aeronautical structures
- Literature survey includes studies on identification of LTI system models
- Intrinsic nature of the excitation may pose difficulties, e.g., presence of periodic inputs originating from rotating components of the structure, in addition to stochastic inputs
- Identified eigenspectrum contains both system and periodic modes
- Consistency of the covariance-based subspace identification for measurements with oscillatory components shown in [1]
- In practice periodic modes often disturb the estimation of close structural modes

AIMS
- Reconstruction of output signal where the periodic part is removed as preprocessing for engineering applications
- Identification of the eigenspectrum of the stochastic system part only

MODELING
- System states are physical quantities of mechanical system i.e. displacements and velocities
- Periodic states represent the periodic excitation $u(t)$

Continuous-time combined state-space model
\[
\begin{align*}
\dot{x}^s(t) &= A^s x^s(t) + Bu(t) + w(t) \\
y(t) &= C^s x^s(t) + Du(t) + v(t)
\end{align*}
\]

Discrete-time combined state-space model
\[
\begin{align*}
\begin{bmatrix} x^{s,0} \\ x^{s,1} \\ \vdots \\ x^{s,n} \end{bmatrix} &= A^s \begin{bmatrix} x^{s,0} \\ x^{s,1} \\ \vdots \\ x^{s,n} \end{bmatrix} + \begin{bmatrix} b^s \\ b^s \\ \vdots \\ b^s \end{bmatrix}w(t) \\
y(t) &= C^s \begin{bmatrix} x^{s,0} \\ x^{s,1} \\ \vdots \\ x^{s,n} \end{bmatrix} + \begin{bmatrix} d^s \\ d^s \\ \vdots \\ d^s \end{bmatrix}v(t)
\end{align*}
\]

\[
A_{E}^{s} \lambda_{E}^{s} \rightarrow Y_{E}^{s} = A_{E}^{s} Y_{E}^{s} + B_{E}^{s} u \quad \perp \quad A_{V}^{s} \lambda_{V}^{s} \rightarrow Y_{V}^{s} = A_{V}^{s} Y_{V}^{s} + B_{V}^{s} u
\]

- Proposed modeling is equivalent to [2] up to similarity transform
- Identification of both system and periodic modes with the UPC algorithm [2]:

\[
\begin{align*}
A_{E}^{s} &= \begin{bmatrix} A_{E}^{1} & \cdots & A_{E}^{n} \end{bmatrix} \\
A_{V}^{s} &= \begin{bmatrix} A_{V}^{1} & \cdots & A_{V}^{n} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
B_{E}^{s} &= \begin{bmatrix} B_{E}^{1} \\ \vdots \\ B_{E}^{n} \end{bmatrix} \\
B_{V}^{s} &= \begin{bmatrix} B_{V}^{1} \\ \vdots \\ B_{V}^{n} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
C_{E}^{s} &= \begin{bmatrix} C_{E}^{1} \\ \vdots \\ C_{E}^{n} \end{bmatrix} \\
C_{V}^{s} &= \begin{bmatrix} C_{V}^{1} \\ \vdots \\ C_{V}^{n} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
d_{E}^{s} &= \begin{bmatrix} d_{E}^{1} \\ \vdots \\ d_{E}^{n} \end{bmatrix} \\
d_{V}^{s} &= \begin{bmatrix} d_{V}^{1} \\ \vdots \\ d_{V}^{n} \end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
E_{E}^{s} &= \begin{bmatrix} E_{E}^{1} \\ \vdots \\ E_{E}^{n} \end{bmatrix} \\
E_{V}^{s} &= \begin{bmatrix} E_{V}^{1} \\ \vdots \\ E_{V}^{n} \end{bmatrix}
\end{align*}
\]

REMOVAL OF THE PERIODIC SUBSIGNAL BY ORTHOGONAL PROJECTION AND SYSTEM IDENTIFICATION

The goal is to reconstruct responses where the periodic signal is discarded and then to identify the observability matrix of the structural system: $\Gamma_{sys}^{\text{sys}} = [C_{E}^{s} (C_{E}^{s} A_{E}^{s})^{T} \cdots (C_{E}^{s} A_{E}^{s})^{T} C_{E}^{s}]$. 

Algorithm:
- Prediction of the periodic subsignal with the non-steady-state Kalman filter

\[
\begin{align*}
\dot{\hat{x}}_{s+1}^E &= A_{E}^{s} \hat{x}^E_{s+1} + B_{E}^{s} u \\
\hat{y}_s &= C_{E}^{s} \hat{x}_s
\end{align*}
\]

with $V = [\Psi^T \Theta^T ]$, where $\Psi = [\psi_1 \cdots \psi_n \psi_{n+1} \cdots \psi_{2n}]$, and $\hat{y}_s^E = V^{-1} \hat{x}_s$.

\[
A_{E}^{s} = V^{T} A_{E}^{s} V, \quad C_{E}^{s} = CV, \quad K_{E}^{s} = V^{T} K_{E}
\]

This allows to select states corresponding to the periodic modes with a selection matrix $\Psi$ and subsequently approximate the periodic subsignal

\[
\hat{y}_s^E = C_{E}^{s} \hat{x}_s^E
\]

CONCLUSIONS
- Extension of the output-only stochastic and periodic state-space modeling to the UPC algorithm
- Consistent algorithm to reconstruct the stochastic part of the system response to random and periodic excitation
- In practical application parameters estimated from the reconstructed measurements are closer to estimates from a comparable random vibration experiment

REFERENCES