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STATISTICAL DAMAGE DETECTION AND LOCALIZATION WITH MAHALANOBIS DISTANCE APPLIED TO MODAL PARAMETERS

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ABSTRACT
Damage detection and damage localization constitute two pillars of Structural Health Monitoring that are highly relevant for applications to large-scale structures. Damage detection is usually achieved through statistical tests of data-driven residuals that monitor changes of a structure from its baseline behaviour. Damage localization investigates changes in damage residuals with respect to parameterized structural models through sensitivity vectors. Among the classic damage-sensitive features used for residual generation are subspace angles and principal components obtained from data spaces, whose evaluation for a decision about damage often boils down to novelty analysis, or statistical likelihood ratio tests. Modal parameter estimates are also employed for this purpose; however, most of the existing approaches appear to neglect the uncertainties related to their estimation. This paper fills this gap and presents a residual for damage detection and damage localization that is based on the difference of modal parameters obtained from data collected in some baseline and some test state of the structural system. The proposed scheme is evaluated in numerical simulations validating its robustness for damage detection and damage localization.

Keywords: damage detection, damage localization, modal parameter estimation, Structural Health Monitoring, Operational Modal Analysis

1. INTRODUCTION

Damage detection and damage localization are two pillars of vibration-based Structural Health Monitoring (SHM) that are well-explored in the literature, e.g., see [1]. Among the many strategies for damage detection and damage localization which are model-based [2, 3], data-driven [4–8], or a combination thereof [9], methods that appreciate the uncertainty in the estimated parameters are particularly appealing from the practical standpoint, as they account for statistical estimation errors related to noise and short data length in the damage diagnosis problem. Such methods include statistical tests on Kalman filter innovations [10], non-parametric change detection tests based on novelty detection [11], or parametric subspace tests [4–6].

Among the classic damage-sensitive features that are often used for the statistical tests about damage are subspace angles and principal components obtained from data spaces. Modal parameter estimates are also employed for this purpose; however, most of the existing approaches appear to neglect the uncertainties related to their estimation. This paper fills this gap and presents a residual for damage detection and damage localization that is based on the difference of modal parameters obtained from data collected in some baseline and some test state of the structure. The statistical properties of the residual are analyzed and are used for a statistical hypothesis test in a metric that boils down to Mahalanobis distance. The proposed scheme is evaluated in numerical simulations validating its robustness for damage detection and damage localization.

2. SYSTEM MODEL AND MODAL PARAMETER PROPERTIES

In this section the parametrized dynamic model for vibration data is introduced, and the statistical properties of modal parameter estimates are recalled.

2.1. System model

Let $\theta \in \mathbb{R}^p$ be a parameter vector that contains p damage-sensitive parameters of the structural elements of interest. This parametrization is defined after the specific monitoring problem at hand, such that θ contains parameters of the dynamic system whose sensitivity to damage is non-zero and which fully parametrize the considered damage, e.g., Young's modulus and density of elements, crack parameters (width, length), among others. The vibration behavior of the monitored linear time-invariant structural system with m degrees of freedom is described by the differential equation

$$\mathcal{M}^\theta \ddot{q}(t) + \mathcal{C}^\theta \dot{q}(t) + \mathcal{K}^\theta q(t) = \mathbf{f}(t) \tag{1}$$

where t denotes continuous time, and $\mathcal{M}^\theta, \mathcal{C}^\theta, \mathcal{K}^\theta \in \mathbb{R}^{m \times m}$ denote mass, damping and stiffness matrices that respectively depend on parameter θ. Vectors $q(t) \in \mathbb{R}^m$ and $\mathbf{f}(t) \in \mathbb{R}^m$ contain the continuous-time displacements at the degrees of freedom (DOF) and the unmeasured external forces, respectively. Let system (1) be observed by sensors measuring accelerations at r DOF of the structure, collected in an output vector $y(t) \in \mathbb{R}^r$

$$y(t) = \mathcal{D}\ddot{q}(t) + \tilde{v}(t), \tag{2}$$

where $\tilde{v}(t) \in \mathbb{R}^r$ denotes the sensor noise and the matrix $\mathcal{D} \in \mathbb{R}^{r \times m}$ selects the acceleration output at the measurement DOF. Sampled at a rate τ, the dynamic behavior of system (1)-(2) can be represented by a discrete-time stochastic state-space model

$$\begin{cases} x_{k+1} = A^\theta x_k + w_k \\ y_k = C^\theta x_k + v_k \end{cases} \tag{3}$$

where $x_k \in \mathbb{R}^n$ are the states, and $A^\theta \in \mathbb{R}^{n \times n}$, $C^\theta \in \mathbb{R}^{r \times n} \in \mathbb{R}^{r \times u}$, are the parametrized state transition and observation matrices estimated at a model order n. Vectors w_k with v_k denote the process and output noises. The eigenfrequencies f_i^θ, damping ratios ζ_i^θ and mode shapes φ_i^θ of the underlying mechanical system are identified for $i = 1 \ldots n$ from the i-th eigenvalue λ_i^θ and eigenvector Φ_i^θ of A^θ such that

$$f_i^\theta = \frac{|\lambda_{ci}^\theta|}{2\pi},\ \zeta_i^\theta = \frac{-\Re(\lambda_{ci}^\theta)}{|\lambda_{ci}^\theta|},\ \varphi_i^\theta = C^\theta \Phi_i^\theta \tag{4}$$

where every eigenvalue of the continuous system λ_{ci}^{θ} is computed with $e^{\lambda_{ci}^{\theta}\tau} = \lambda_i^{\theta}$. The $|(\cdot)|$ denotes the modulus operator and $\Re(\cdot)$ and $\Im(\cdot)$ express the real and imaginary parts of a complex variable.

2.2. Statistical properties of modal parameter estimates

Hereafter assume that $1 \ldots m$ available estimates of modal parameters are consistent, i.e., the estimates converges to their true values when the data length N goes to infinity. Moreover, assume that the estimates of the natural frequencies and the real and the imaginary parts of the estimated mode shapes

$$\hat{z} = \begin{bmatrix} \hat{f}_1^{\theta} & \ldots \hat{f}_m^{\theta} & \Re(\hat{\varphi}_1^{\theta})^T & \ldots & \Re(\hat{\varphi}_m^{\theta})^T & \Re(\hat{\varphi}_1^{\theta})^T & \ldots & \Im(\hat{\varphi}_m^{\theta})^T \end{bmatrix}^T \tag{5}$$

are jointly asymptotically Gaussian, satisfying

$$\hat{z} \approx \mathcal{N}\left(z,\ \tfrac{1}{N}\Sigma_z\right),$$

where

$$z = \begin{bmatrix} f_1^{\theta} & \ldots & f_m^{\theta} & \Re(\varphi_1^{\theta})^T & \ldots & \Re(\varphi_m^{\theta})^T & \Im(\varphi_1^{\theta})^T & \ldots & \Im(\varphi_m^{\theta})^T \end{bmatrix}, \tag{6}$$

and $\Re(\cdot)$ and $\Im(\cdot)$ express the real and the imaginary parts of a complex variable, $\mathcal{N}(\mu, \Sigma)$ denotes a Gaussian distributed variable with mean μ and covariance Σ, and $\Sigma_z \in \mathbb{R}^{m(2r+1)\times m(2r+1)}$ is the joint asymptotic covariance of the natural frequency and the mode shape estimates. Multiple system identification methods satisfy the aforementioned criteria, e.g., stochastic subspace identification (SSI) methods. The computation of a consistent estimate $\widehat{\Sigma}_z$ can be obtained with the statistical delta method [12] and can be found e.g. in [13–15].

3. MODAL PARAMETER-BASED DAMAGE DETECTION AND LOCALIZATION

Based on features extracted from measurement data in the baseline (reference) and in the current test state, the goal of damage detection is to evaluate whether there is a significant change between the states or not. On the other hand, the overall goal of damage localization is to determine the location of the detected damage based on an FE model of the considered structure and its vibration response collected after the damage occurs. While the detection and the localization of damage relate to different engineering problems, both can be scoped to monitoring changes in the system parameter θ. To analyse such changes, the local approach framework is used [16], where the close hypotheses are formulated

$$\begin{aligned} &\mathrm{H}_0 : \theta = \theta_* \ \text{(healthy state)}, \\ &\mathrm{H}_1 : \theta = \theta_* + \delta/\sqrt{N} \ \text{(damaged state)}, \end{aligned} \tag{7}$$

where δ is an unknown change vector. A data-driven damage residual, whose design is the subject of this work, is formulated in a way such that a small change of the (unknown) θ from its assumed nominal value θ_* induces a change therein. Damage detection refers then to monitoring changes in θ via monitoring changes in the expected value of the residual. The damage localization problem boils down to a statistical decision about which entry of θ is linked to the deviation of the residual from its nominal behaviour.

3.1. Residual definition

Let $\hat{\zeta}$ denote a data-driven damage diagnosis residual. In this work a residual based on the difference of modal parameters is used; in principal, however, any data-driven and damage-sensitive Gaussian metric can be adopted for this purpose, e.g., see [4–7]. To define the residual, let $\hat{z}^{\mathrm{ref}}$ denote stacked estimates of natural frequencies and the vectorized estimates of real and imaginary parts of the mode shapes obtained from data collected in some baseline state of the system, and let $\hat{z}^{\text{test}}$ be its counterpart obtained from data collected in some currently tested system state. The modal parameter-based residual is written as

$$\hat{\zeta} = \sqrt{N}\left(\hat{z}^{\text{ref}} - \hat{z}^{\text{test}}\right). \tag{8}$$

The statistical distribution of the residual (8) can be approximated as Gaussian for a sufficiently large data length N thanks to the asymptotic local approach to change detection [16]

$$\text{under } \mathrm{H}_0 : \hat{\zeta} \xrightarrow{\mathcal{L}} \mathcal{N}(0, \Sigma_\zeta), \tag{9}$$

$$\text{under } \mathrm{H}_1 : \hat{\zeta} \xrightarrow{\mathcal{L}} \mathcal{N}(\mathcal{J}^z_{\theta_*}\delta, \Sigma_\zeta), \tag{10}$$

where $\mathcal{J}^z_{\theta_*} = \frac{\partial z}{\partial \theta}\big|_{\theta=\theta_*}$ is the sensitivity of the modal parameters w.r.t. the system parameter evaluated at θ_*, and $\Sigma_\zeta = \Sigma_{z^{\text{ref}}} + \Sigma_{z^{\text{test}}}$ is the residual covariance that accounts for the uncertainty of both reference and test modal parameters. The derivative $\mathcal{J}^z_{\theta_*}$ is obtained based on the FE model of the mechanical system and is of shape

$$\mathcal{J}^z_{\theta_*} = \begin{bmatrix} \mathcal{J}^f_\theta \\ \Re\left(\mathcal{J}^\varphi_\theta\right) \\ \Im\left(\mathcal{J}^\varphi_\theta\right) \end{bmatrix}, \tag{11}$$

where

$$\mathcal{J}^f_\theta = \begin{bmatrix} \frac{\partial f_1}{\partial \theta^1} & \cdots & \frac{\partial f_1}{\partial \theta^p} \\ \vdots & & \vdots \\ \frac{\partial f_m}{\partial \theta^1} & \cdots & \frac{\partial f_m}{\partial \theta^p} \end{bmatrix}\Bigg|_{\theta=\theta_*}, \quad \mathcal{J}^\varphi_\theta = \begin{bmatrix} \frac{\partial \varphi_1}{\partial \theta^1} & \cdots & \frac{\partial \varphi_1}{\partial \theta^p} \\ \vdots & & \vdots \\ \frac{\partial \varphi_m}{\partial \theta^1} & \cdots & \frac{\partial \varphi_m}{\partial \theta^p} \end{bmatrix}\Bigg|_{\theta=\theta_*}. \tag{12}$$

Its computation can be performed analytically [17], or by using, e.g., a finite difference approach.

3.2. Damage detection and localization strategy

The Generalized Likelihood Ratio (GLR) test is used to decide between two distribution functions (9)-(10), which boils down to a weighted Mahalanobis distance

$$t = \check{\zeta}^T \mathcal{A}^{-1}\check{\zeta}, \tag{13}$$

where $\check{\zeta} = \hat{\mathcal{J}}^T\Sigma_\zeta^{-1}\hat{\zeta}$, $\mathcal{A} = \hat{\mathcal{J}}^T\Sigma_\zeta^{-1}\hat{\mathcal{J}}$ and $\hat{\mathcal{J}}$ is a consistent estimate of $\mathcal{J}^z_{\theta_*}$. For a decision about the damage, the test value (13) is compared to a threshold corresponding to a quantile of the theoretical distribution of the reference test statistics. Notice that a parameter-free version of (13) can be formulated by assuming that $\mathcal{J}^z_{\theta_*}$ is the identity matrix of appropriate size.

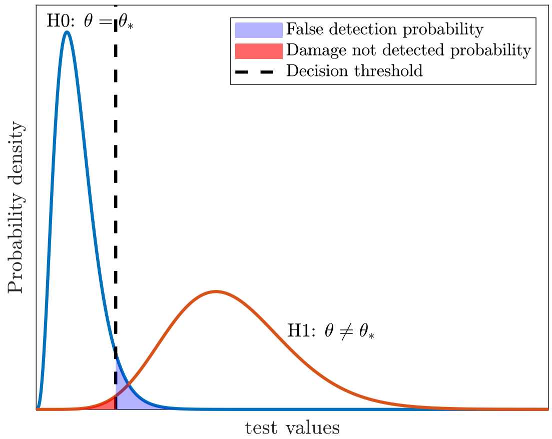
To locate damage, it has to be decided which entries of θ_* possibly have changed, which boils down to testing each j-th entry of the change parameter δ, i.e., $\delta^j = 0$ (no damage) against $\delta^j \neq 0$ (indicating damage), which yields the test statistic [9]

$$t_j = \check{\zeta}_j^{\,T} (\mathcal{A}^j)^{-1}\check{\zeta}_j, \tag{14}$$

where $\check{\zeta} = \hat{\mathcal{J}}_j^T\Sigma_\zeta^{-1}\hat{\zeta}$, $\mathcal{A}^j = \hat{\mathcal{J}}_j^T\Sigma_\zeta^{-1}\hat{\mathcal{J}}_j$ and $\hat{\mathcal{J}}_j = \hat{\mathcal{J}}^z_{\theta_*^j}$ is a column of $\hat{\mathcal{J}}^z_{\theta_*}$ corresponding to the sensitivity of the residual w.r.t. to change of the j-th parameter. The test statistics t_j is asymptotically χ^2 distributed with $d = 1$ degrees of freedom and non-centrality parameter λ_j

$$\lambda_j = \mathcal{A}^j(\delta^j)^2 \tag{15}$$

if $\delta^j \neq 0$ and the other entries of δ are null. When the latter assumption is not satisfied, e.g., when damaged pertains to more than one element, or when testing an undamaged element while others are damaged, the non-centrality parameter does not follow (15) and a minmax test to locate damage should be used [9]. The sketch of the deployed damage detection and localization tandem is presented in Figure 1.



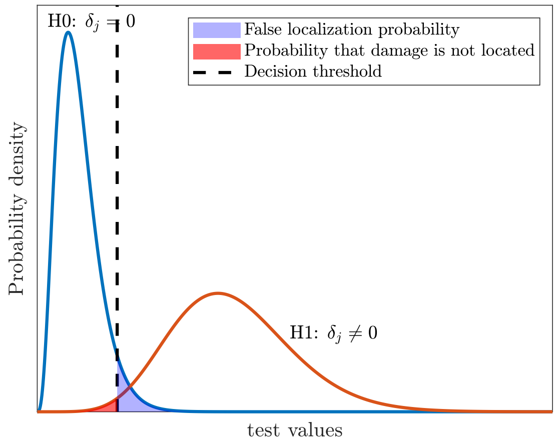


Figure 1: Damage detection scheme (left). Damage localization scheme (right).

Remark 1. *The mode shape contained in* $\hat{z}$ (5) *is called the unnormalized mode shape since its scaling is arbitrary. To make it comparable between different data sets a normalization scheme is needed. On the related note, the covariance of the normalized mode shapes is rank deficient [18], which must be considered in* (13)-(14) *by removing the adequate rows. Two normalization methods were analyzed in detail in [18], from which the normalization with the maximum mode shape component is used herein.*

4. APPLICATION

This section is devoted to the application of the proposed damage detection and localization scheme on data simulated based on Reissner-Mindlin plate model. The model consists of 64 first-order elements, 81 nodes and, consequently, 243 degrees of freedom (DOF). For the sake of simulation, proportional damping is assumed, where the damping matrix is defined such that each mode has a damping ratio of 1%. The translational DOF are constrained to zero at the edge elements of the model. The excitation is modeled as a white noise signal applied on all DOFs. The transverse acceleration data are sampled with a frequency of 6000 Hz and collected with 11 sensors. One damage scenario emulating a 15% increase in mass of element 47 is considered. The plate model is depicted in Figure 2.

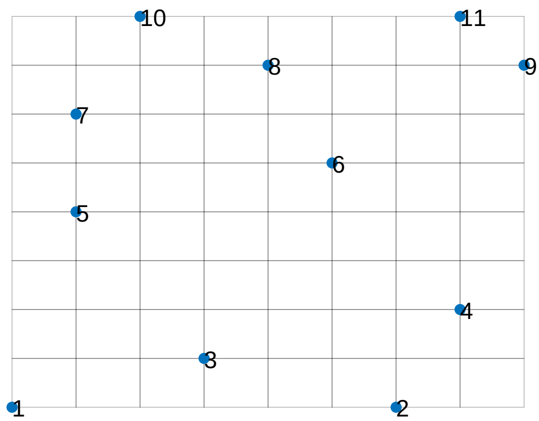


Figure 2: Plate model with sensors.

In total, 200 data sets of length N = 1,000,000 in both healthy and damage states are simulated. For each data set, modal parameters corresponding to the first 8 bending modes of the plate and their corresponding covariance are estimated with stochastic subspace identification and the first-order delta method [14]. The estimates of natural frequencies tracked across the simulated data are shown in the left part of Figure 3.

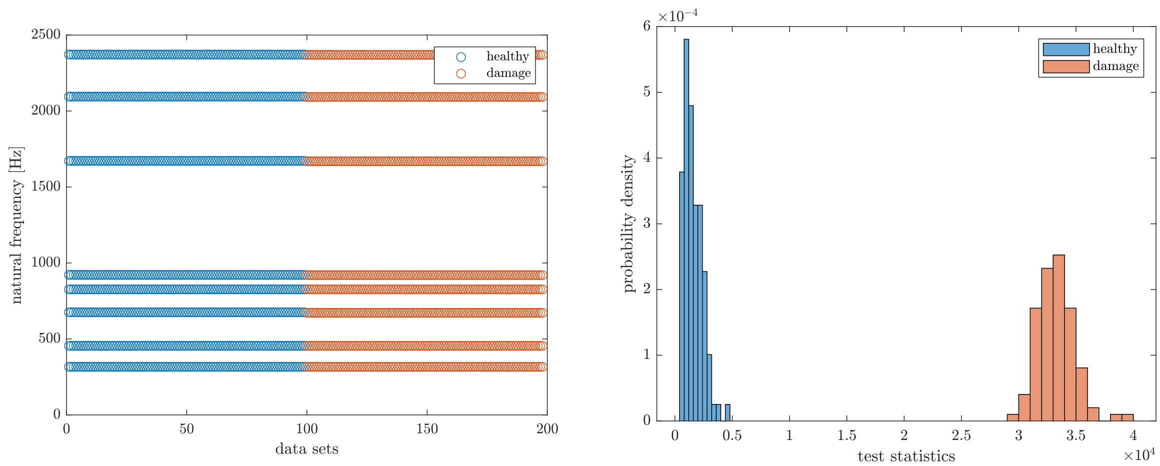


Figure 3: Natural frequency estimates obtained from both healthy and damage data (left). Histograms of damage detection test (right).

It can be viewed that for every data set the complete set of 8 modes is estimated and that the change in the natural frequency estimates due to damage can hardly be distinguished from the visual inspection. To detect damage, the non-parametric modal parameter-based test, i.e., when assuming that $\mathcal{J}^z_{\theta_*}$ is an identity matrix, is employed. The distribution of the test statistics is illustrated in Figure 3 (right). The inflicted damages are clearly detected and a clear separation between safe and damaged states is observed.

Subsequently, the localization of the mass change in element 47 is considered. Due to a limited number of sensors compared to the large FE model-based parametrization θ, the sensitivity of the residual with respect to some components of θ may be equal, or be very close. Thus, such parameter components are indistinguishable, and clustering of parameters in $\mathcal{J}^z_{\theta_*}$ is performed. For this purpose the hierarchical complete-linkage clustering of the normalized residual sensitivity after [9, 19] is used, and 26 parameter clusters are distinguished. A dendogram diagram showing the clustered parameters is illustrated in Figure 4 and the left part of Figure 5. Subsequently, the minmax damage localization test is performed with using the centers of the clustered sensitivity matrix. The test results are shown in the right part of Figure 5.

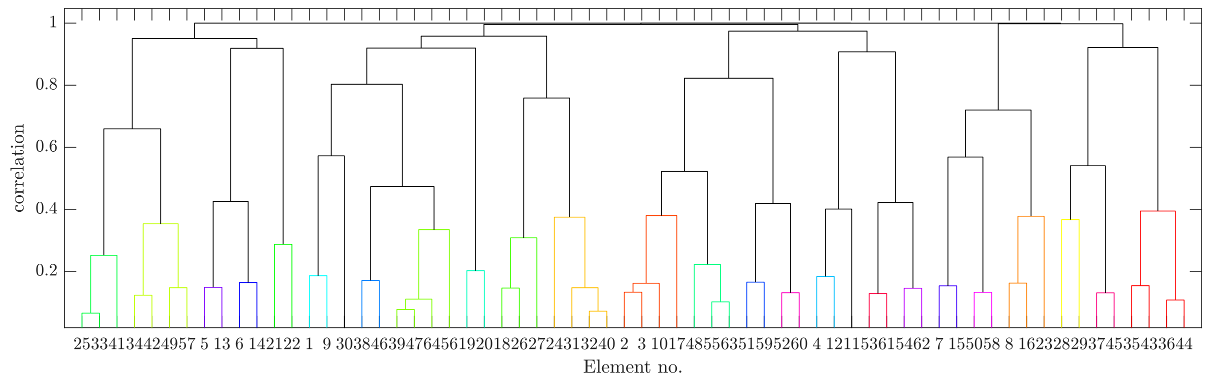


Figure 4: Dendogram showing hierarchical complete-linkage clustering of modal parameter sensitivities.

It can be viewed that the damage localization test yields the highest score for the cluster containing the damaged element, and it can be clearly distinguished from the values of the test corresponding to the remaining clusters.

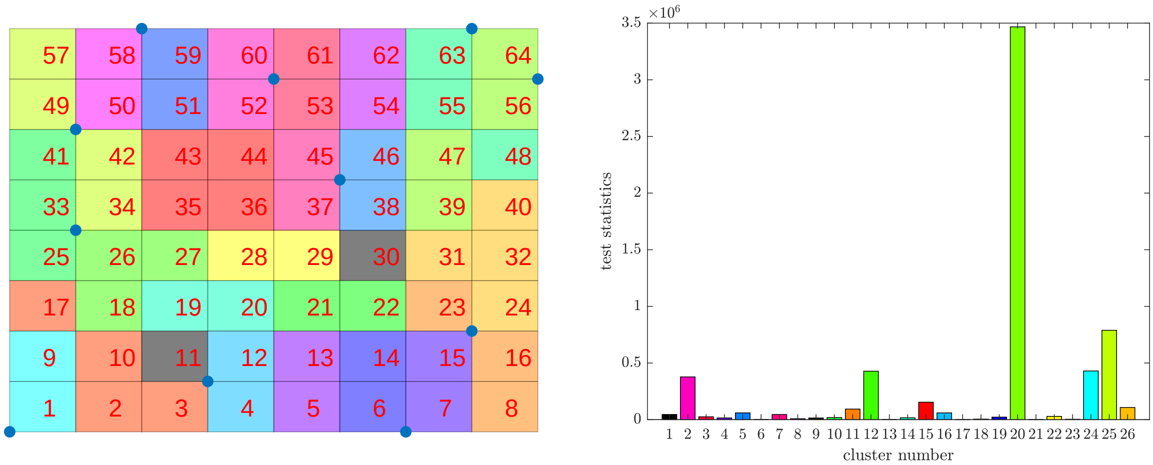


Figure 5: Plate element clusters (left). Damage localization results - damage in element 47, corresponding to the 20th cluster (right).

5. CONCLUSIONS

In this paper, a Gaussian residual for damage detection and damage localization has been derived based on the Mahalanobis distance between modal parameters in different states of the structure. The capabilities of the method were showcased on a plate model, where a small change in the density of one element were clearly detected, and localized in a clustered parameter space.

REFERENCES

[1] O. Avci, O. Abdeljaber, S. Kiranyaz, M. Hussein, M. Gabbouj, D. J. Inman, A review of vibration-based damage detection in civil structures: From traditional methods to machine learning and deep learning applications, Mechanical Systems and Signal Processing 147 (2021) 107077.

[2] D. Bernal, Damage Localization from the Null Space of Changes in the Transfer Matrix, AIAA Journal 45 (2007) 374–381.

[3] M. D. Ulriksen, D. Bernal, L. Damkilde, Shaped input distributions for structural damage localization, Mechanical Systems and Signal Processing 110 (2018) 499–508.

[4] M. Döhler, L. Mevel, F. Hille, Subspace-based damage detection under changes in the ambient excitation statistics, Mechanical Systems and Signal Processing 45 (1) (2014) 207 – 224.

[5] E. Viefhues, M. Döhler, F. Hille, L. Mevel, Statistical subspace-based damage detection with estimated reference, Mechanical Systems and Signal Processing 164 (2022) 108241.

[6] S. Greś, M. Döhler, P. Andersen, L. Mevel, Subspace-based mahalanobis damage detection robust to changes in excitation covariance, Structural Control and Health Monitoring 28 (8) (2021) e2760.

[7] Y. Ou, E. N. Chatzi, V. K. Dertimanis, M. D. Spiridonakos, Vibration-based experimental damage detection of a small-scale wind turbine blade, Structural Health Monitoring 16 (1) (2017) 79–96.

[8] S. Laflamme, L. Cao, E. Chatzi, F. Ubertini, Damage detection and localization from dense network of strain sensors, Shock and Vibration 2016 (2016).

[9] S. Allahdadian, M. Döhler, C. Ventura, L. Mevel, Towards robust statistical damage localization via model-based sensitivity clustering, Mechanical Systems and Signal Processing 134 (2019) 106341.

[10] D. Bernal, Kalman filter damage detection in the presence of changing process and measurement noise, Mechanical Systems and Signal Processing 39 (1-2) (2013) 361–371.

[11] K. Worden, G. Manson, N. Fieller, Damage detection using outlier analysis, Journal of Sound and Vibration 229 (3) (2000) 647 – 667.

[12] G. Casella, R. L. Berger, Statistical Inference, 2nd Edition, Cengage Learning, 2001.

[13] E. Reynders, R. Pintelon, G. De Roeck, Uncertainty bounds on modal parameters obtained from stochastic subspace identification, Mechanical Systems and Signal Processing 22 (4) (2008) 948 – 969.

[14] M. Döhler, L. Mevel, Efficient multi-order uncertainty computation for stochastic subspace identification, Mechanical Systems and Signal Processing 38 (2) (2013) 346–366.

[15] P. Mellinger, M. Döhler, L. Mevel, Variance estimation of modal parameters from output-only and input/output subspace-based system identification, Journal of Sound and Vibration 379 (C) (2016) 1 – 27.

[16] A. Benveniste, M. Basseville, G. Moustakides, The asymptotic local approach to change detection and model validation, IEEE Transactions on Automatic Control 32 (7) (1987) 583–592.

[17] W. Heylen, S. Lammens, P. Sas, et al., Modal analysis theory and testing, Vol. 200, Katholieke Universiteit Leuven Leuven, Belgium, 1997.

[18] S. Greś, M. Döhler, P. Andersen, L. Mevel, Uncertainty quantification for the Modal Phase Collinearity of complex mode shapes, Mechanical Systems and Signal Processing 152 (2021) 107436.

[19] A. Mendler, M. Döhler, C. Ventura, L. Mevel, Clustering of redundant parameters for fault isolation with Gaussian residuals, in: 21st IFAC World Congress, 2020.